Sandstone Compaction Modelling and Reservoir Simulation

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Summary

Soil compaction has a great influence on reservoir pressure development and hydrocarbon production by depletion. Still this factor is often treated in a simplistic manner in reservoir simulation studies, mainly due to lack of data in the early stages of production history. Later in field history the compaction parameters can often be satisfactorily estimated by material balance analysis, but for field development feasibility studies this option is obviously not available.
In this paper a general model for behaviour of sand or sandstones under compaction is presented. If necessary rock parameters are not available, it is shown how reasonable estimates can be deducted from petrophysics. Available methodology for compaction modelling in reservoir simulators is shown to be at best only approximate, and can be a source of grave errors. A systematic approach to improved compaction modelling is presented.

Introduction

The most common scheme for hydrocarbon production is such that average reservoir pressure is decreasing with time, at least in the early field history. In this phase the sand pore volume is generally reduced by soil compaction, which is the most important energy supply during depletion. Simulated reservoir pressure development and fluid production is an important input factor for determining e.g. necessary injection support and water handling capacity. Such factors have a direct influence on economical break-even analyses, which in the worst case may lead to erroneous field development decisions.
In the exploration phase, available data for rock strength parameters are typically scarce or lacking, whereby the simulations often are carried through with some generic value for rock compressibility. Figure 1 shows a series of runs where the compressibility was varied from 100 MPa (very soft sand) to 50 GPa (extremely hard sandstone). Even half an order of magnitude change in compressibility has great impact on the results, but the uncertainty in compressibility is frequently larger than that in real case studies.

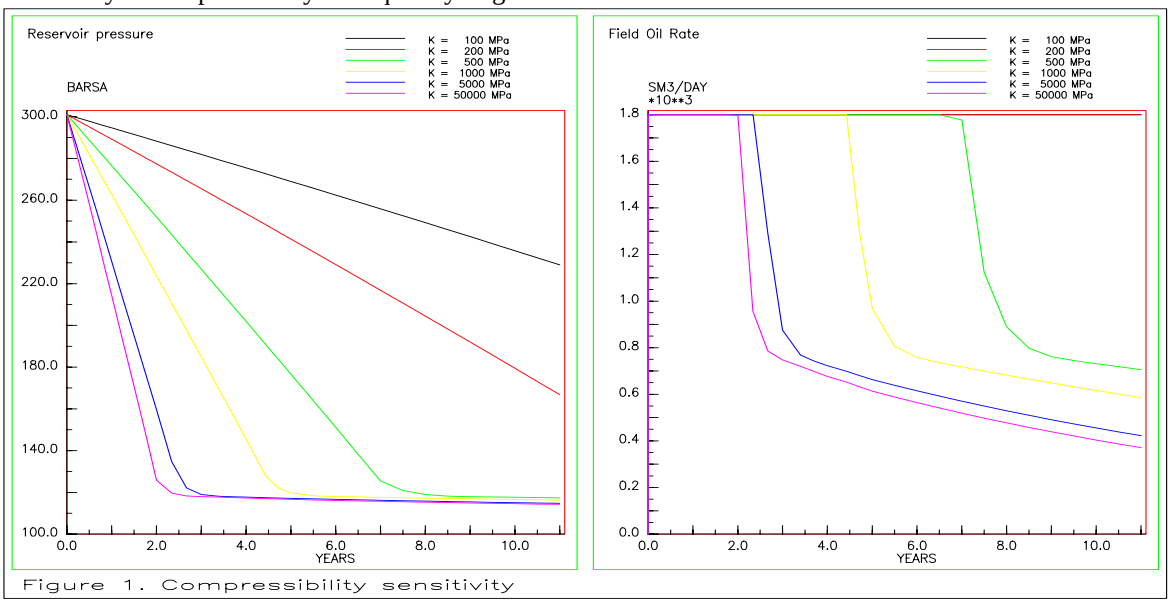

Figure 1. Compressibility sensitivity

Consequentially, rock parameter data should be provided prior to the simulation studies, but even when the data is available it is not obvious how to use it in simulation. It is unavoidable that some studies are carried through with insufficient data, and in such cases rules of thumb for defining compaction behaviour are needed. These subjects are the focus of this paper.

The Grain Packing Model

Compressibility is defined as

$$K = \frac{E}{3(1-2\nu)}$$, where E is Young's modulus and ν Poisson's ratio.

A control volume V_B is comprised of a volume V_S of solid (grains) and a pore volume V_P.
The *specific volume* v is the ratio of bulk to solids volumes, and it is easily seen that

$$v = \frac{V_B}{V_S} = \frac{1}{1-\phi}$$, where ϕ is porosity.

In sand or sandstones the grains will typically be quartz, for which $K_S \approx 38$ GPa. Typical measured bulk volume compressibility K_B range from 100 MPa to 1 GPa for sands, and from 5 to 15 GPa for sandstones.
Hence K_S is considerably larger than K_B, for sands a couple of orders of magnitude larger. Clearly, measured bulk compression cannot be explained by grain compression alone, but must be a consequence of reordering of the sand grains or / and changes in grain bonding agent structure, which is not discussed in this paper.
Biot's constant is defined as

$$\alpha = 1 - \frac{K_B}{K_S}$$ (e.g. Fjær et al., 1992). For sands, α is often set to unity, neglecting grain compression.

Since bulk volume compaction is primarily governed by the pore volume compaction, we can set up the following presuppositions,

- Compaction implies grain packing reorganization
- The grain packing is determined by the historically maximal stress state
- For a given stress state, the grains will tend to settle at the most stable packing pattern
- Tighter grain packing requires larger confining forces to achieve further packing than less tight packing
- The packing will maintain its most stable state, i.e. during subsequent unloading the grains will not return to a previous (less stable) packing state. (Irreversible process)
- Fracturing is probable under high (deviatoric) stress

For modelling purposes the implications are that primary compaction should be considered as a permanent (irreversible) pore space deformation. Only during a secondary unloading – loading cycle can elastic behaviour be expected.

Rock Mechanic Compaction Model

We propose a simple model for compaction which is consistent with the presuppositions above and fits experimental data reasonably well,

$$K(p) = K_0 + a(p - p_0) + b(p - p_0)^2$$,

where p is mean effective stress, p_0 is an initial value of p, and $K_0 = K(p_0)$.
The curve-fitting parameters a and b should satisfy,

$a > 0$; to ensure hardening under compaction
$b > 0$; to ensure further compaction is increasingly harder to achieve
a and b depend on K_0 such that when K_0 increases, so do a and b.

Certainly measured data should be used when available, but in lack of experimental $K(p)$, reasonably good estimates of K_0, a, and b can be obtained from sand strength and porosity using similarity analysis.

From the definitions of compressibility and volumetric strain ε_p, we can derive

$$v(p) = v(p_0)\exp(-\int_{p_0}^{p} \frac{dp}{K(p)}); \quad v(p_0) = \frac{1}{1-\phi}$$,

where p is mean effective stress (Wood, 1990).

Typical variation of specific volume vs. mean effective stress according to this expression is shown in the upper half of Figure 2, marked "iso-ncl". Note that the curve is only valid for a first time loading process, i.e. increasing p.

This model is an example of Critical State Theory (Wood, 1990).
By this theory, specific volume will be characterized by

- Primary loading defined by the isotropic normal compression line (iso-ncl)
- Subsequent unloading and secondary loading defined by unloading – reloading lines (url)
- Each state on the iso-ncl corresponds to a point on the current yield surface in the p:q-plane (q is deviatoric stress)

- Changes of v within the yield surface are purely elastic, while reduction of v on the iso-ncl is determined by expansion of the current yield surface, which is governed by the *hardening rule*

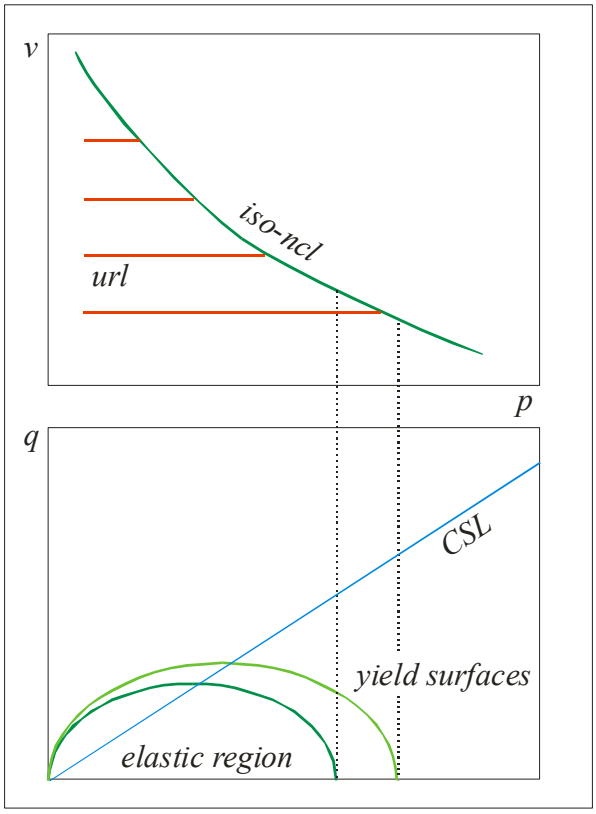

Figure 2.

$$\frac{\partial p}{\partial \varepsilon_p} = p\frac{v}{v_0}H$$

A special case of critical state theory is the Cam Clay Model, where it is assumed that

- the iso-ncl is a straight line in the log(p):q plane
- the yield surfaces are ellipses.
- For the discussed model, the horizontal half axis is always the present value of p
- The vertical half axis is defined by the critical state line angle, which can be evaluated from the Mohr-Coulomb friction angle

The iso-ncl and hardening parameters are defined as

$$v = v_\lambda - \lambda \log p; \quad H = \frac{v_0}{\lambda}$$

where λ is the inclination of the iso-ncl, which can be determined by

$$\lambda = \frac{v(p_2) - v(p_1)}{p_2 - p_1},$$

where p1 and p2 are two "representative values" of mean effective stress.

This model, which is a "standard" failure model in soil mechanics, is also a direct consequence of the grain packing model. It was therefore used in the rock mechanics simulations to compute stress and strain distributions from which the "correct" compaction could be derived.

Compaction modelling in reservoir simulation

Probably the most used compaction model in simulation is the linear elastic, where only a constant compressibility coefficient is used. This model is incompatible with the analysis above.
A more general approach is to define a pore volume multiplier m, as a (tabulated) function of fluid pressure P, $m = m(P; P_0) = \frac{V_P(P)}{V_P(P_0)} = \frac{v(P)}{v(P_0)}$. This function can be irreversible, whereby the pore volume in any cell will always be at its historical minimum, or it can be defined with hysteresis. The function $m(P; P_0)$ can (and should) be made consistent with the cam clay theory by computing the m-values from the iso-ncl.
However, the iso-ncl is a function of mean effective stress p, while m is a function of fluid pressure P.
By the definition of effective stress, $\sigma' = \sigma - \alpha P$, where σ and σ' are total and effective stress, and α is Biot's constant. Hence, *the use of fluid pressure in lieu of mean effective stress when computing m is valid only if the total stress is constant*.
To study the validity of the flow simulator compaction model, $m(p; p_0)$ was derived from an iso-ncl, and the function was used naïvely by replacing p with P. Next the corresponding stress field was computed by the finite element rock mechanics simulator Visage™ from V.I.P.S Ltd., London.

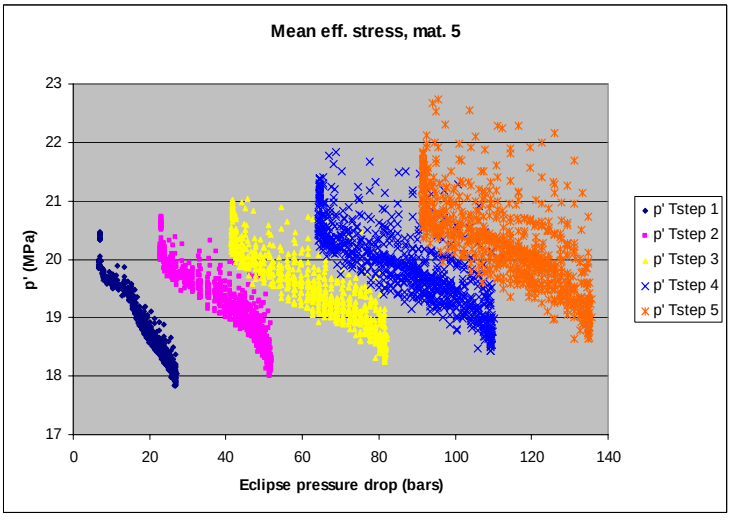

Figure 3

Visage can be run in coupled rock-mechanics – flow-simulator mode with pore volume correction, such that at each stress computation step the calculations are iterated until a consistent solution w.r.t fluid pressure, stress state, and compaction has been found. The Visage finite element grid extends far beyond the reservoir grid such that the fixed boundary has small influence on reservoir stress.
If the flow simulator compaction computations are valid, mean effective stress should be a linear function of fluid pressure. A number of simulations were performed, with a wide range of sand strengths and with varying degree of reservoir heterogeneity. *None of the results could support the validity of the flow simulator compaction model*. A typical result is shown in Figure 3 (fluid pressure drop has been used, and compressive stress is positive). Although the reservoir is heterogeneous, only results from a single material type is shown. For this example it is clear that actual, detailed compaction distribution cannot be computed from fluid pressure alone, and it appears that the full coupled flow – rock mechanics simulations must be carried through to achieve this.
Specific volume can be computed from (simulated) strain state from the definition of strain:

$$\frac{\delta v}{v} = -\delta\varepsilon_p \Rightarrow v(p) = v_0 \exp\left[\varepsilon_p(p_0) - \varepsilon_p(p)\right]$$

By this expression we can compute pore volume multipliers from the Visage computed strain and compare with those from the flow simulator. Such a correlation is shown in Figure 4, which clearly shows that the two methods are far from equivalent.

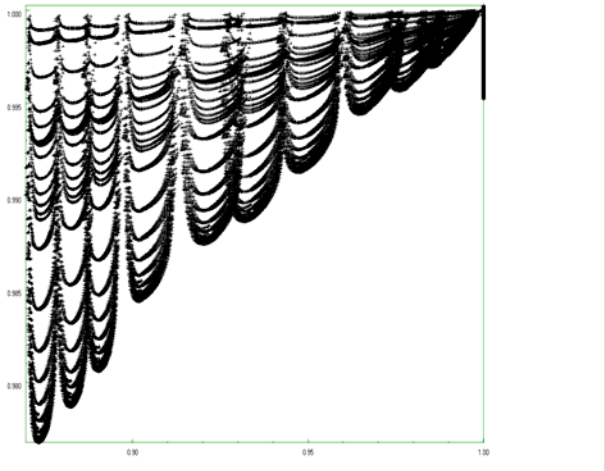

Figure 4. PVmult from strain vs. from P

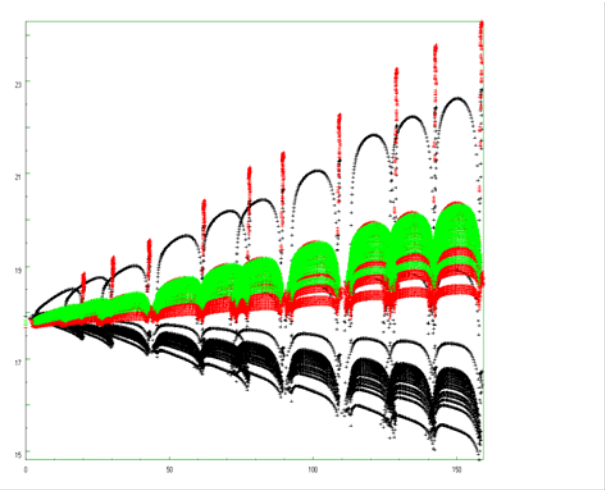

Figure 5. Mean stress vs. fluid pressure

For a homogeneous reservoir with a single material, trends are more easily established. In Figure 5, mean stress vs. *P* has been plotted for 10 stress time steps,

- for all grid cells (black – red – green)
- omitting the bottom layer (red – green)
- omitting all cells at the reservoir edge (green)

Confining the volume of interest to non-edge grid cells only (green points) produces a reasonably good correlation. It is apparent that the lack of trend is to a large degree caused by cells near material edges, and the bottom layer (black points) especially contributes to irregular behaviour due to uplift of the underburden. For this case, where the offending cells can be identified, the flow simulator compaction calculations can be improved on by redefining the *m*-functions for edge cells.
This concept can be generalized such that derived *m*-functions are used only away from material edges, and modified (interpolated) *m*-functions near such edges. A heterogeneous reservoir (Figure 3), has internal material boundaries almost everywhere, which explains why no systematic trends could be detected.
In ongoing work we aim to systemize the isolation of trend regions such that redefinition of the *m*-functions is done automatically by special purpose software, which communicates with both simulators and update relevant data between iteration cycles. By this approach we expect to be able to generate flow simulator *m*-functions which are compatible with the rock mechanics model, and can be used to run similar cases in stand alone flow simulator mode.

References:

- Fjær, E., Holt, R.M., Horsrud, P., Raaen, A.M., Risnes, R., 1992. *Petroleum Related Rock Mechanics,* Developments in Petroleum Science, 33, Elsevier.
- Wood, D.M., 1990. *Soil Behaviour and Critical State Soil Mechanics.* Cambridge University Press