Sandstone compaction, grain packing and Critical State Theory

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ABSTRACT: Based on the physics of grain packing in a granular material, this paper demonstrates that sands or sandstones are modelled most correctly by Critical State Theory, which can be used to define a consistent compaction relationship for use in rock mechanics or reservoir simulation. The theoretical model is compared with experimental data for volume and permeability variation during loading or unloading.

KEYWORDS: *compaction, failure model, reservoir simulation, rock mechanics, sandstone*

INTRODUCTION

An understanding of the influence of stress on porosity and permeability is of importance in geomechanics applications in the geosciences and reservoir engineering. During recent decades there has been a growing awareness that the dynamic stress state in the reservoir and surrounding rock can have a significant influence on the reservoir exploitation scheme through interaction between the stress field and petrophysical parameters, for example – an interaction that is best understood by performing rock mechanics simulations or coupled rock mechanics–flow simulations. The geomechanical simulation model must be based on a poro-elasto-plastic model for soil behaviour which should reproduce actual soil behaviour as closely as possible. Linear elasticity is often assumed and the most popular failure model seems to be the Mohr–Coulomb criterion. This paper will demonstrate that these may not always be the most appropriate choices for sands or sandstones. The focus will be on grain behaviour during compaction.

A number of papers have been published on this subject, falling mainly into three groups: experimental work; theoretical models for grain interaction; and grain movement simulation.

Obviously, the experimental results should be the primary source for establishing stress–strain relationships during loading, and classification of dependency, for example, on mean versus shear stress, and mechanisms leading to dilation or failure. Schutjens (1991) studied compaction and creep of dry and saturated quartz sands, producing volumetric strain vs. mean effective stress. Omar *et al.* (2003) performed experiments on a number of granular soil samples from the United Arab Emirates, and classified grain properties and compaction characteristics. Zhu *et al.* (1997) investigated the influence of radial stress on porosity and permeability on sandstone by triaxial extension tests to failure. Experiments leading to a classification of permanent deformation/failure and characterizing of yield surfaces for sandstones were performed by Nihei *et al.* (2000) and Karner *et al.* (2003).

Theoretical micromechanical models for grain interaction were studied by Brandt (1955), White (1983), Schwartz (1984) and Zimmerman (1991). A good overview can be found in Fjær *et al.* (1992). Canals & Meunier (1995) presented a mathematical model for porosity reduction by quartz cementation during compaction. Herrmann (1999) provided an example of theoretical models for powders. Such models give a good understanding of grain packing, but are primarily valid for low confining pressure conditions and, as such, may fall outside this paper's scope of interest.

Numerical experiments of grain organization during compaction and shearing contribute to the understanding of these processes, although they are generally limited to idealized materials. Examples of studies performed using the discrete element method are Sitharam & Nimbkar (2000), Guo & Morgan (2004) and Morgan (2004).

This paper takes a somewhat different approach. An attempt will be made to describe compaction in a granular material (idealized sand/sandstone) by a simple, yet convincing, theoretical model, based on fundamental physical principles. The model is principally in agreement with experimental and simulated results reported in the referenced papers and identifies Critical State Theory as the most appropriate poro-elasto-plastic model for such materials.

NOTATION

The **bulk modulus** for a material is defined as

$$K = \frac{E}{3(1 - 2\nu)}, \tag{1}$$

where E is Young's Modulus and ν the Poisson coefficient. The inverse $1/K$ is then a measure for volumetric compression. For a porous material distinction is made between the bulk modulus for a bulk volume (solids and pore space) K_B, and the solids (grain) bulk modulus K_S.

Compaction in a porous material is dependent on effective stress σ, which is related to total stress σ_T and fluid pressure p_f by

$$\sigma = \sigma_T - \alpha p_f \tag{2}$$

where Biot's constant α is defined as

$$\alpha = 1 - \frac{K_B}{K_S} \tag{3}$$

(Biot 1941; Terzaghi 1943; Wood 1990). A control volume is denoted by V_B, V_S and V_P, respectively for bulk volume, solids volume and pore volume. Then porosity ϕ is the ratio of pore volume to bulk volume, and **specific volume** v,

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$$v = \frac{V_B}{V_S} = \frac{1}{1 - \frac{V_P}{V_B}} = \frac{1}{1 - \phi} \quad (4)$$

The effective stress σ and total stress σ_T are symmetrical 3×3 tensors, with components $\sigma=\{\sigma_{xx},\sigma_{yy},\sigma_{zz},\sigma_{xy},\sigma_{xz},\sigma_{yz}\}$ for σ, and similar for σ_T. For the strain tensor ε a similar notation will be used.

The mean effective stress is $p=\frac{\sigma_{xx}+\sigma_{yy}+\sigma_{zz}}{3}$ and volumetric strain $\varepsilon_p=\varepsilon_{xx}+\varepsilon_{yy}+\varepsilon_{zz}$.

The **deviator stress** q is understood most easily as the difference between the axial and radial stress, valid for a cylinder-symmetric sample. The general definition is more complex (see e.g. Wood 1990),

$$q = \left\{ \frac{(\sigma_{xx} - \sigma_{yy})^2 + (\sigma_{xx} - \sigma_{zz})^2 + (\sigma_{yy} - \sigma_{zz})^2}{2} + 3(\sigma_{xy}^2 + \sigma_{xz}^2 + \sigma_{yz}^2) \right\}^{1/2} \quad (5)$$

THE (IDEALIZED) GRAIN PACK MODEL

The basis for the grain pack model is the observation that bulk compressibility for sands/sandstones is much larger than grain compressibility. Indeed, typical magnitudes are:

- $K_S \approx 38$ GPa (quartz grains);
- K_B(sand)=0.1–1 GPa;
- K_B(sandstone)=5–15 GPa.

Hence, grain compaction contributes insignificantly to the bulk compaction, especially for sands.

For sands, and many sandstones, Biot's constant is often set to unity, which is equivalent to assuming that $K_B \ll K_S$, an assumption that is justified by the numerical values above.

In the following, for simplicity, $\alpha=1$, but the arguments will hold equally well for α not too far from 1, by rephrasing such terms as 'rigid' with 'almost rigid', etc.

Obviously, for a skeleton of rigid grains to compact, the entire compaction must be attributed to pore volume reduction. This is, however, impossible to achieve without reorganizing the grains, hence each level of compaction corresponds to some grain packing configuration (Morland & Sawicki 1983).

In addition, the physical principle of **stable settlement** is used: when grains reorganize they will always tend to seek the most stable packing pattern available and never reconfigure from an existing packing to a less stable one. Taking these two principles as granted, some interesting consequences can be inferred directly.

- In a loading process, each (effective) stress state corresponds to a stable packing configuration, the tightest possible packing at that stress level.
- The soil has no memory of its previous states and a compressing soil can equally well be regarded as changing, such that each packing level defines a 'new' material with its own poro-elasto-plastic parameters. At pore level, this process is seen as continuous pore wall failure during loading.
- As packing becomes tighter, further packing will be increasingly more difficult to achieve, and each packing level is more stable than previous levels. This implies that K_B should be increasing with p.
- Relieving stress will not return the soil to a previous, less stable packing level. Hence, the soil is permanently deformed by compaction (plasticity) (Morland & Sawicki 1983) and the present grain packing is a result of the historical maximum p. Thereby, typical reservoir soils are not maximally packed at initial conditions, since further packing would be impossible.
- In a reservoir under loading conditions, all local reservoir volumes will contract or remain unchanged. Hence, the bulk reservoir volume is reduced, and this reduction must be compensated by expansion or movement of over-, under-, and side-burdens – some degree of subsidence or overburden swelling should be expected for sandstone reservoirs.

Note that only pure packing compaction has been discussed. Real soils will also be subjected to other, complicating mechanisms, such as those listed below.

- During a load increase the soil may fracture instead of having a tighter packing. This will be seen as a sudden reduction in strength. (Indeed, apparent constant K_B is often a process where strength increase is followed by fracturing in rapid succession (Wood 1990).) A related behaviour is that grain particle corners may break off during reorganization (Zhu *et al.* 1997; Chuhan *et al.* 2003).
- The soil is not 'pure'. The void space may be partly filled with bonding agents and/or fine-grained material which may break or dissolve during flooding. The fines can settle in the pore space or be transported by flowing fluid (Canals & Meunier 1994).
- Shear stress may cause dilation in place of or in addition to compaction (Wood 1990; Karner *et al.* 2003).

Such effects are not part of the grain pack model, but should be considered separately and included as a modification of the basic model. Intuitively, these mechanisms should strengthen the validity of the consequences noted above rather than weaken them (fracturing excepted).

For a primary loading process in the pore compression regime, it is proposed that the dependency on effective stress for the bulk modulus can be expressed as,

$$K(p) = K_0 + a(p - p_0) + b(p - p_0)^2 \quad (6)$$

where index 0 is used to denote a state where no load is present. (Obviously, equation (6) ceases to be valid if bulk compaction becomes comparable to grain compaction.)

In order to be compatible with the statements above,

1. one expects $a > 0$, to ensure hardening (tighter packing) when p increases;
2. a positive b signifies accelerated hardening as packing becomes tighter;
3. a and b should depend on the initial bulk modulus K_0 and such that two different compressibility curves $K^{(1)}$ and $K^{(2)}$ satisfy $K_0^{(1)} < K_0^{(2)} \Rightarrow a^{(1)} < a^{(2)}$.

Based on experiments on soils from six North Sea sandstone reservoirs (experiments performed by Edinburgh Rock Mechanics Consortium, confidential report), it was found that equation (6) can fit accurately most of the measurements, and that statements 1 and 3 above are, indeed, satisfied for almost all the samples. Figures 1 and 2 show $K(p)$ for relative strong and weak sandstones respectively (measured data and polynomial approximation). The strong sandstones behave as expected, with K increasing with load. The rapid strength increase followed by a sudden decrease seen in Figure 2 was observed in many of the experiments on weak samples and can be explained as follows. Initially, the sample is weak, with a relatively loose grain packing. Hence, only a small increase in load is needed to increase the packing density considerably, resulting in a

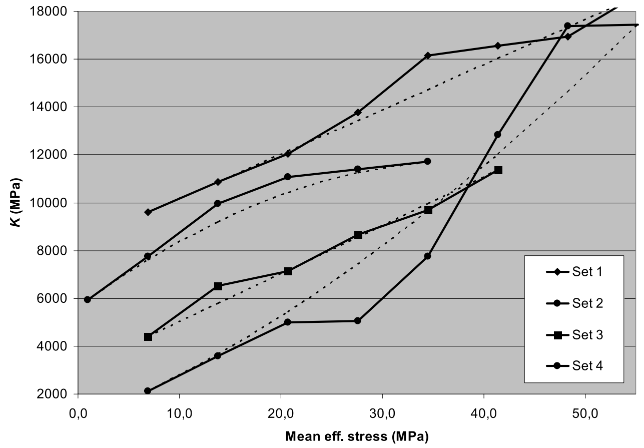

Fig. 1. $K(p)$, relative strong sandstone. Dashed lines: corresponding polynomial approximation.

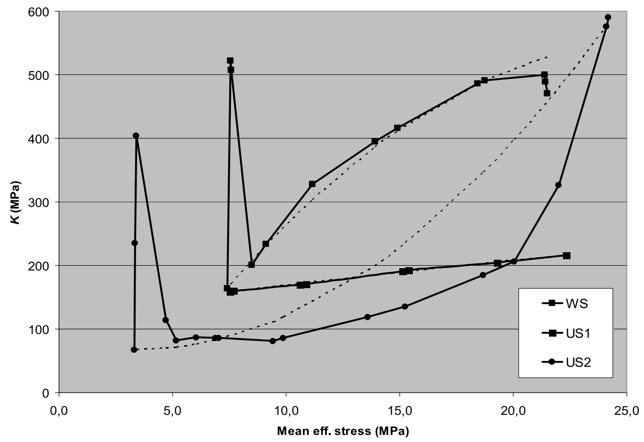

Fig. 2. $K(p)$, weak/unconsolidated sandstone.

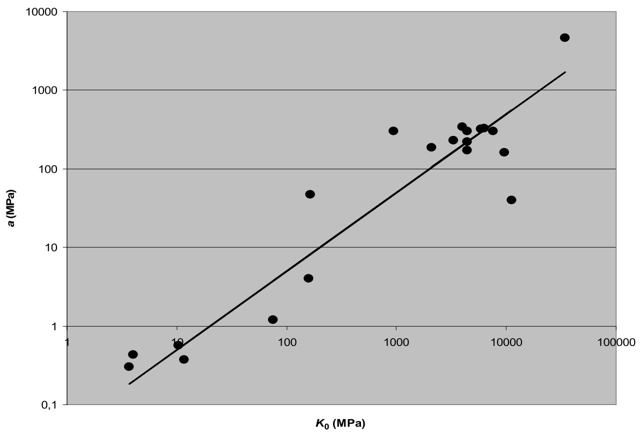

Fig. 3. Coefficient a vs. no-load bulk modulus K_0. Correlation line: a=0.05K_0.

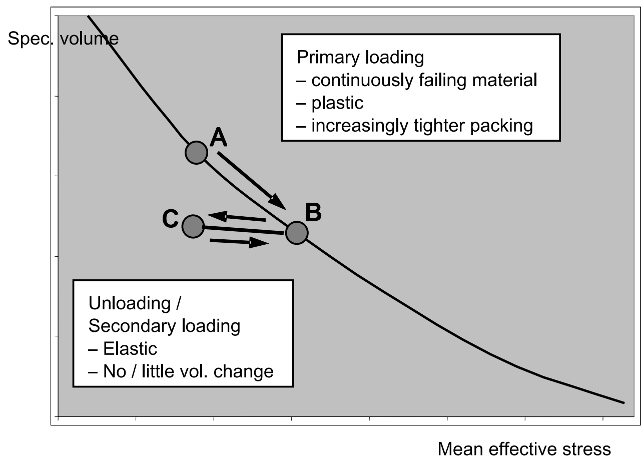

Fig. 4. Schematic of grain pack compaction.

significant increase in soil strength. On continued load increase the sample fractures by shear failure and, at that stage, a more stable packing configuration has been achieved, such that compaction by further loading will be in agreement with the expected hardening behaviour.

An interesting question is whether the polynomial approximation can be used in a more general setting, i.e. if it is possible to determine generic constants a and b such that equation (6) can be used to determine compressibility behaviour from other reservoir parameters when no or few experimental soil strength data are available. To that extent, Figure 3 shows a correlation between the coefficient a and initial compressibility K_0. It can be seen that a=0.05K_0 gives a reasonably good match. (The figure is also a strong indicator of the validity of assumption 3.) A similar correlation for coefficient b did not, however, show any systematic behaviour, indicating no preference to accelerated or retarded hardening. Hence, if a generic b must be used, it is probably best to set it to zero.

In conclusion, measured data should be used when available, but can be replaced with the polynomial approximation to gain the advantage of smooth data. The generic polynomial (a=0.05K_0, b=0) with K_0 determined from porosity, for example, can be of acceptable quality, but should generally be used only when no measured data are available.

From the definitions of compressibility and volumetric strain one has (Wood 1990)

$$\frac{\delta\varepsilon_p}{\delta p} = \frac{1}{K}\ and\ \delta\varepsilon_p = -\frac{\delta v}{v} = -\delta log v \tag{7}$$

whereby

$$v(p) = v(p_0)exp\left(-\int_{p_0}^{p}\frac{dp}{K(p)}\right);\ v(p_0) = \frac{1}{1-\phi_0} \tag{8}$$

Using equations (8) and (4), $v(p)$ and $V_B(p)$ can be computed.

The characteristics of the grain pack model are summarized in Figure 4, which shows the variation of specific volume during primary loading (A→B), unloading (B→C) and secondary loading (C→B).

The main features are:

- during primary loading the material is continuously failing (plasticity), corresponding to expansion of the yield surface in p:q space;
- unloading/secondary loading is elastic, with small or no change in specific volume.

Such a $v \leftrightarrow p$ dependency is exactly as formulated by the Critical State Theory (Wood 1990; Goulty 2004). Hence, for the discussed type of materials, Critical State Theory is the appropriate failure model to use.

Referring to Figure 5, in the p:v space, primary loading is along the isotropic normal compression line (iso-ncl), while unloading and secondary loading follow the unloading/reloading lines (url). Decrease of v on the iso-ncl corresponds to expansion of the current yield surface in the p:q space, determined by the hardening rule,

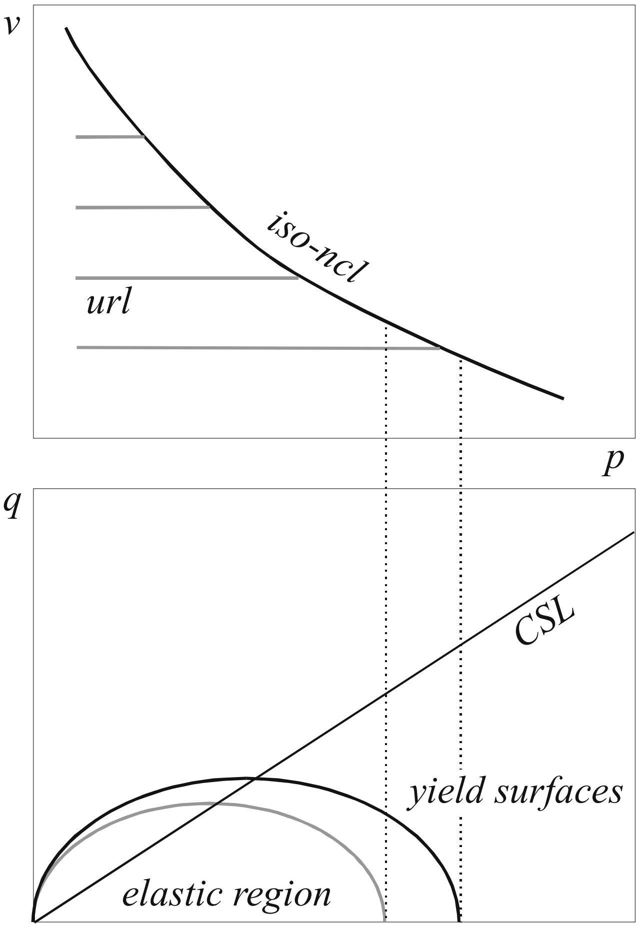

Fig. 5. Specific volume and yield surfaces in Critical State Theory. iso-ncl, isotropic normal compression line; url, unloading/reloading lines; CSL, critical state line.

$$\frac{\partial p}{\partial \varepsilon_p} = p\frac{v}{v_0}H \tag{9}$$

where H is the hardening parameter. Change of v along urls occur in the elastic region interior to the yield surface in p:q space.

Since the actual shape of the yield surface is nearly impossible to measure, it is common practice to use a specialization of Critical State Theory, namely the **Cam Clay Model** (Wood 1990), where

- the iso-ncl is a straight line in the log(p):v plane,

$$v = v_\lambda - \lambda log(p) \tag{10}$$

- the yield surfaces are ellipses with the major axis along the p-axis. For the grain pack model the length of the horizontal ellipse axis will be the current value of p. The vertical ellipse axis is determined by the critical state line (CSL on Figure 5), which can be found from the friction angle;
- the hardening parameter is

$$H = \frac{v_0}{\lambda} \tag{11}$$

The parameter λ can be determined from the specific volume curves, as

$$\lambda = \frac{v(p_2) - v(p_1)}{log(p_2) - log(p_1)} \tag{12}$$

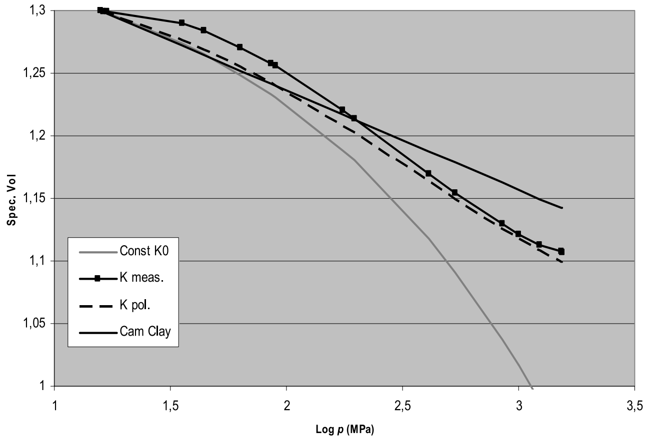

Fig. 6. Specific volume $v(p)$, weak sandstone.

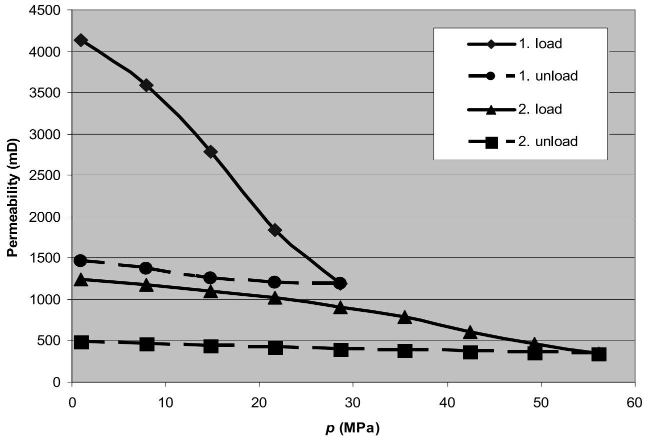

Fig. 7. Permeability vs. p during repeated load/unload.

where p_1 and p_2 are chosen such that the resulting iso-ncl fits the data as good as possible in the relevant effective stress range. The yield surfaces (ellipses) and hardening parameter define the failure model in a rock mechanics simulator, while specific volume curves (or rather pore volume multiplier curves) derived from these are used as to define compaction vs. fluid pressure in a reservoir simulator for doing coupled simulations.

As an example, Figure 6 shows specific volumes computed from measured data, the polynomial approximation, and the Cam Clay Model. For comparison the linear elastic model (K constant) is also shown. The appropriate parameters, from curve-fitting and equations (11), (12), are, a=1.2, b=0.4, λ=0.079, H=16.

CONSEQUENCES FOR PERMEABILITY

By the mechanisms of grain packing one would expect permeability variation with load to be qualitatively similar to the variation of specific volume and this is, indeed, what has been observed in most experiments. Figure 7 provides an example (weak sandstone). An increasingly tighter grain packing during loading implies that the rate of permeability loss with load would be greatest when the initial permeability is high, i.e. permeability differences in heterogeneous soils will tend to be reduced by loading. Figure 8 supports this assumption: from permeability vs. load experiments, the rate of permeability change ($\frac{dk}{dp}$), versus initial (no-load) permeability k_0 has been computed.

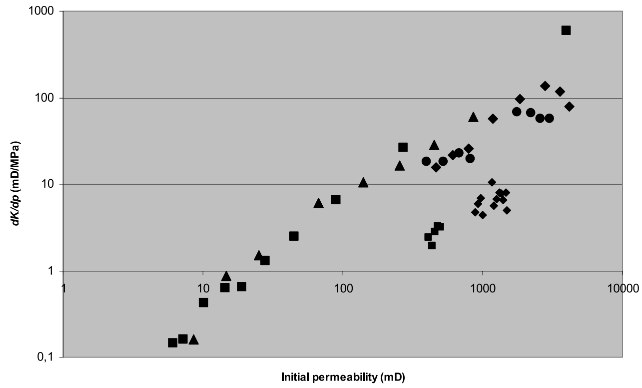

Fig. 8. Permeability rate of change during load vs. no-load permeability.

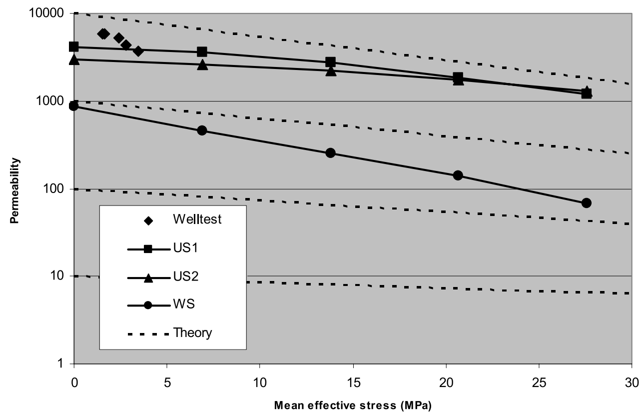

Fig. 9. Permeability vs. load, measured and theory.

A simple permeability model with the desired qualitative variation is,

$$log(k) = \left(1 - \frac{p}{p^*}\right) log(k_0) \tag{13}$$

p^* is a (large) value of p, such that $k(p^*)=1mD$, irrespective of the initial value k_0. Figure 9 shows some measured permeability vs. load data, compared to curves corresponding to equation (13), with $p^*=150$ *MPa*. Sands/sandstones having a qualitative behaviour of this kind will tend to have permeability contrasts reduced during load. In some cases this can contribute to improved oil recovery, as in a fluvial reservoir, for example, where injection water will preferentially flow through the high permeable channels, bypassing large oil volumes in the low permeable background sands. By reducing fluid pressure and, hence, mean effective stress, the contrast between channel and background permeabilities is reduced (homogenization) and a more uniform sweep pattern can result (Standnes *et al.* 2005).

CONCLUSIONS

By using fundamental physical principles of grain packing it has been demonstrated that Critical State Theory is the appropriate poro-elasto-plastic model to use for sands or sandstones. The theoretical considerations are in good agreement with measured data. Idealized analytical models were presented for compaction and permeability behaviour under loading conditions, and consistent compaction functions for use in rock mechanics and reservoir simulators were developed.

REFERENCES

Biot, M.A. 1941. General Theory of three dimensional consolidation. *Journal of Applied Physics,* **12**, 155–163.

Brandt, H. 1955. A study of the speed of sound in porous granular media. *Journal of Applied Mechanics,* **22**, 479–486.

Canals, M. & Meunier, J.D. 1995. A model for porosity reduction in quartzite reservoirs by quartz cementation. *Geochimica et Cosmochimica Acta,* **59**, 699–709.

Chuhan, F.A., Kjeldstad, A., Bjørlykke, K. & Høeg, K. 2003. Experimental Compression of Loose Sands: Relevance to Porosity Reduction During Burial in Sedimentary Basins. *Canadian Geotechnical Journal,* **40**, 995–1011.

Fjær, E., Holt, R.M., Horsrud, P. & Raaen, A.M. 1992. *Petroleum Related Rock Mechanics.* Elsevier Science Publishers B. V., The Netherlands.

Goulty, N.R. 2004. Mechanical compaction behaviour of natural clays and implications for pore pressure estimation. *Petroleum Geoscience,* **10**, 73–79.

Guo, Y. & Morgan, J.K. 2004. Influence of normal stress and grain shape on granular friction: Results of discrete element simulations. *Journal of Geophysical Research,* **109**, B12305.

Herrmann, H.J. 1999. Statistical models for granular materials. *Physica A,* **263**, 51–62.

Karner, S.L., Chester, F.M., Kronenberg, A.K. & Chester, J.S. 2003. Subcritical compaction and yielding of granular quartz sand. *Tectonophysics,* **377**, 357–381.

Morgan, J.K. 2004. Particle dynamics simulations of rate and state-dependent frictional sliding of granular fault gouge. *Pure and Applied Geophysics,* **161**, 1877–1891.

Morland, L.W. & Sawicki, A. 1983. A mixture model for the compaction of saturated sand. *Mechanics of Materials,* **2**, 203–216.

Nihei, K.T., Hilbert, L.B. Jr., Cook, N.G.W., Nakagawa, S. & Myer, L.R. 2000. Frictional effects on the volumetric strain of sandstone. *International Journal of Rock Mechanics and Mining Sciences,* **37**, 121–132.

Omar, M., Shanableh, A., Basma, A. & Barakat, S. 2003. Compaction characteristics of granular soils in United Arab Emirates. *Geotechnical and Geological Engineering,* **21**, 283–295.

Schutjens, P.T.M. 1991. Experimental compaction of quartz sand at low effective stress and temperature conditions. *Journal of the Geological Society, London,* **148**, 527–539.

Schwartz, L.M. 1984. Acoustic properties of porous systems: II. Microscopic description. *In:* Johnson, D.L. & Sen, P.N. (eds) *Physics and Chemistry of Porous Media.* American Institute of Physics, New York, 105–118.

Sitharam, T.G. & Nimbkar, M.S. 2000. Micromechanical Modelling of Granular Materials: Effect of Particle Size and Gradation. *Geotechnical and Geological Engineering,* **18**, 91–117.

Standnes, D. C., Skauge, A. & Pettersen, Ø. 2005. Effects to be Considered When Planning Late Stage Depressurisation. Paper presented at the 13th European Symposium on Improved Oil Recovery, Budapest, Hungary, 25–27 April.

Terzaghi, K. 1943. *Theoretical Soil Mechanics.* Wiley, New York.

White, J.E. 1983. *Underground Sound.* Elsevier, New York.

Wood, D.M. 1990. *Soil Behaviour and Critical State Soil Mechanics.* Cambridge University Press, Cambridge.

Zhu, W., Laurent, G.J.M. & Wong, T. 1997. Shear-enhanced compaction and permeability reduction: Triaxial extension tests on porous sandstone. *Mechanics of Materials,* **25**, 199–214.

Zimmerman, R.W. 1991. *Compressibility of Sandstones.* Elsevier Science Publishers B.V., Amsterdam.

Received 4 July 2005; revised typescript accepted 28 April 2006.