# Rock Mechanics Seminar Series 2010

# Bonus 2: Challenges in Coupled Simulations

# Flow equations $F(p_f, S_w) = 0$

Without loss of generality we study isothermal two-phase flow neglecting capillary pressure. Further we assume the solid is incompressible.

$$\nabla\left\{\frac{kk_{rl}}{\mu_l}(\nabla p_f - \gamma_l \nabla z)\right\} = \frac{\partial}{\partial t}\left(\frac{\phi S_l}{B_l}\right) + q_l, \quad l = o, w$$

$$S_o + S_w = 1$$

| | |
|---|---|
| $k$: | Absolute permeability |
| $k_{rl} = k_{rl}(S_w)$: | Relative permeability |
| $\mu_l = \mu_l(p_f)$: | Viscosity |
| $p_f$: | Fluid pressure |
| $\gamma_l$: | Gravity term |
| $\phi$: | Porosity |
| $S_l$: | Saturation |
| $B_l$: | Volume factor |
| $q_l$: | Source term |

# Rock mechanics equations $R(\varepsilon, \sigma, u) = 0$

Stress - strain relationship :

$$\Delta\sigma_{ij} = D_{ijkl}(\varepsilon_{kl} - \varepsilon^{p}_{kl}) - \Delta p_f \delta_{ij}$$

Plastic flow rule :

$$\delta\varepsilon^{p}_{ij} = \delta\lambda \frac{\partial g}{\partial \sigma_{ij}}$$

Consistency :

$$dF(\mathbf{J}, \chi) = 0$$

| | |
|---|---|
| $\boldsymbol{\sigma}$: | (Eff.) stress tensor |
| $\boldsymbol{\varepsilon}$: | Strain tensor |
| $\mathbf{D}$: | Elastic constants |
| $(\cdot)^p$: | Plastic |
| $\lambda$: | Plastic multiplier |
| $g$: | Plastic potential |
| $F()$: | Yield surface |
| $\mathbf{J}$: | Stress invariants |
| $\chi$: | Hardening parameter |
| $\mathbf{u}$: | Displacement |

Stress invariants: Many variations; the ones used by Visage:

Mean stress : $$J_1 = p = \frac{1}{3}(\sigma_1 + \sigma_2 + \sigma_3)$$

Deviatoric stress : $$J_2 = \frac{q}{\sqrt{3}} = \sqrt{\frac{(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2}{6}}$$

Lode's angle : $$J_3 = \theta = \tan^{-1}\left[\frac{1}{\sqrt{3}}\left(2\frac{(\sigma_2 - \sigma_3)}{(\sigma_1 - \sigma_3)}\right) - 1\right] \qquad -30^\circ \le \theta \le 30^\circ$$

# The coupling term

When the flow equations are solved as a stand-alone problem, it is normally assumed that rock compressibility is a function of pressure:

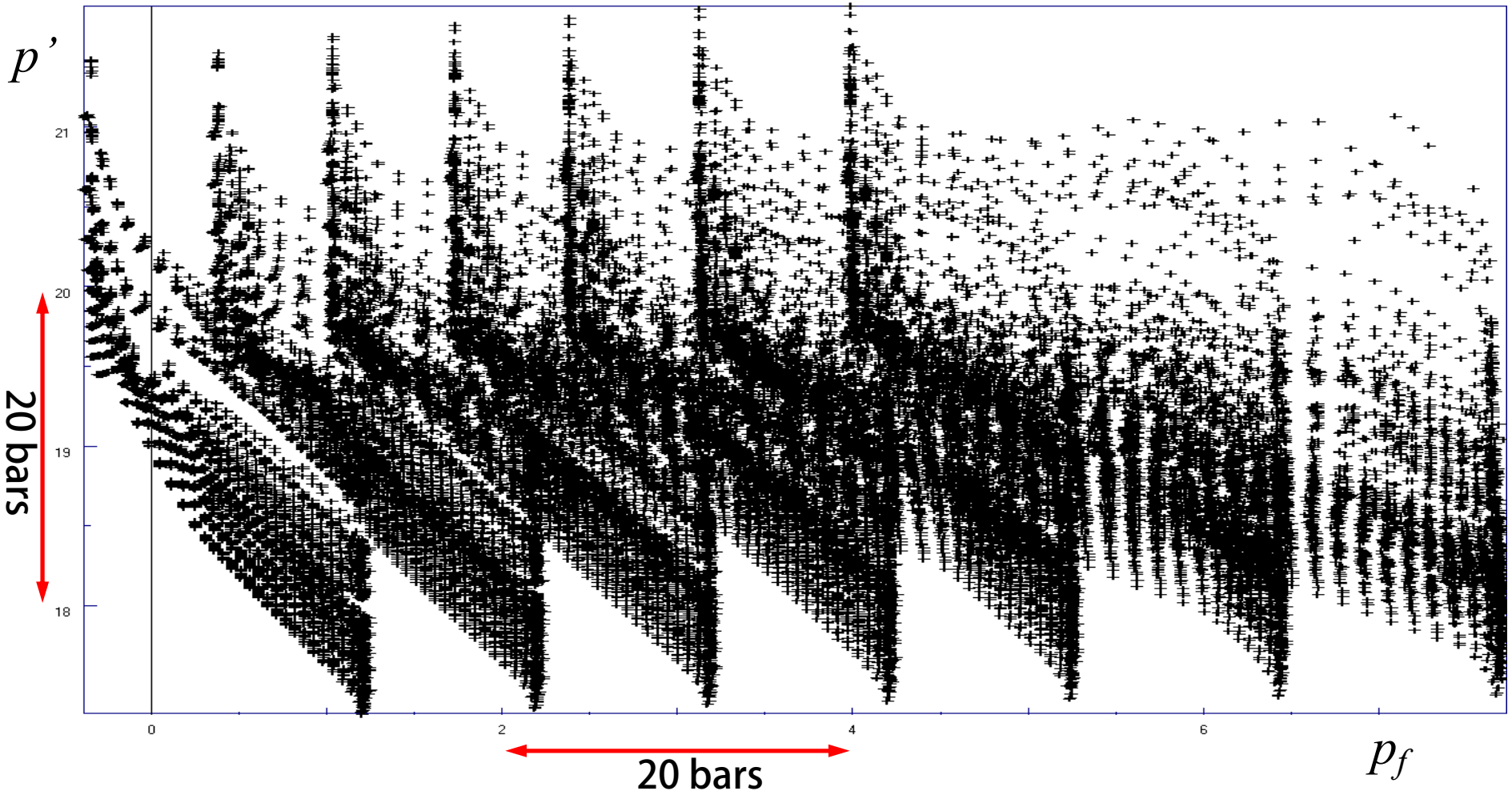
$$c_r(p_f) = -\frac{1}{V}\frac{\partial V}{\partial p_f}$$

This is a natural approach, as rock compressibility in most cases is too important to neglect, and $p_f$ is the only relevant parameter available in the flow equations.

As we saw earlier, pore volume reduction is determined by the net force acting on the pore walls, i.e. *the effective stress*.

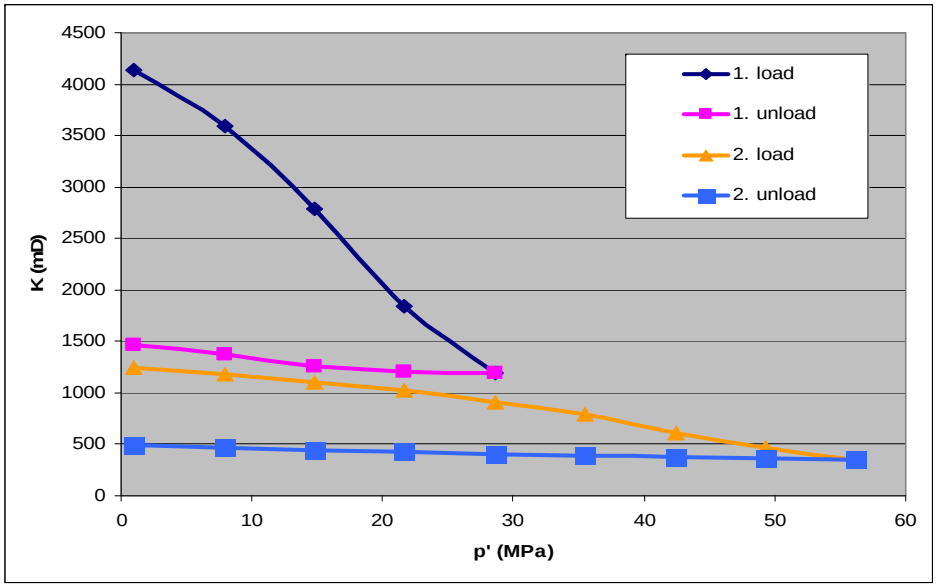
Hence, the pressure-dependency is only valid if $p$' is proportional to $p_f$, which is very seldom the case (example next slide).

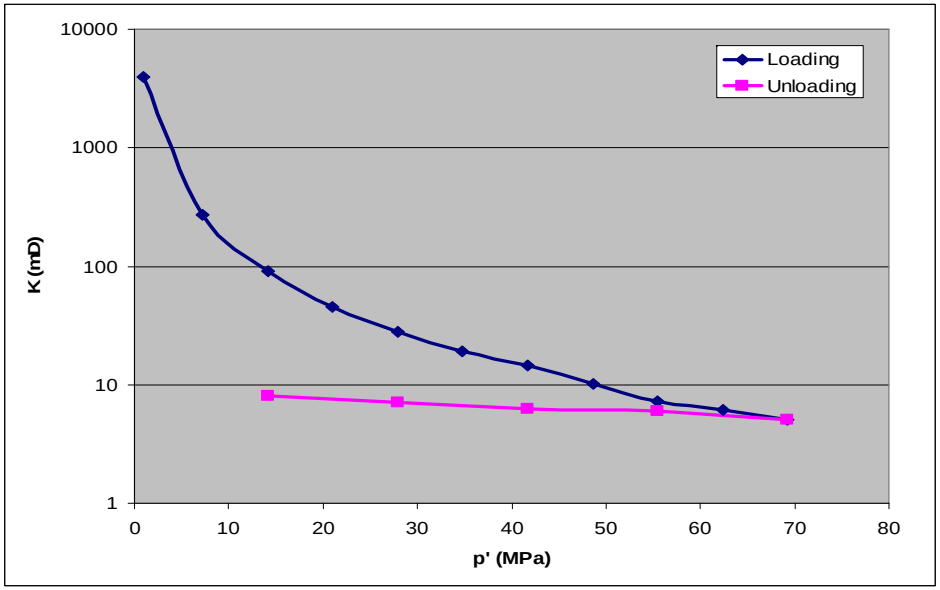
# Correlation: Mean Eff. Stress vs. Fluid Pressure

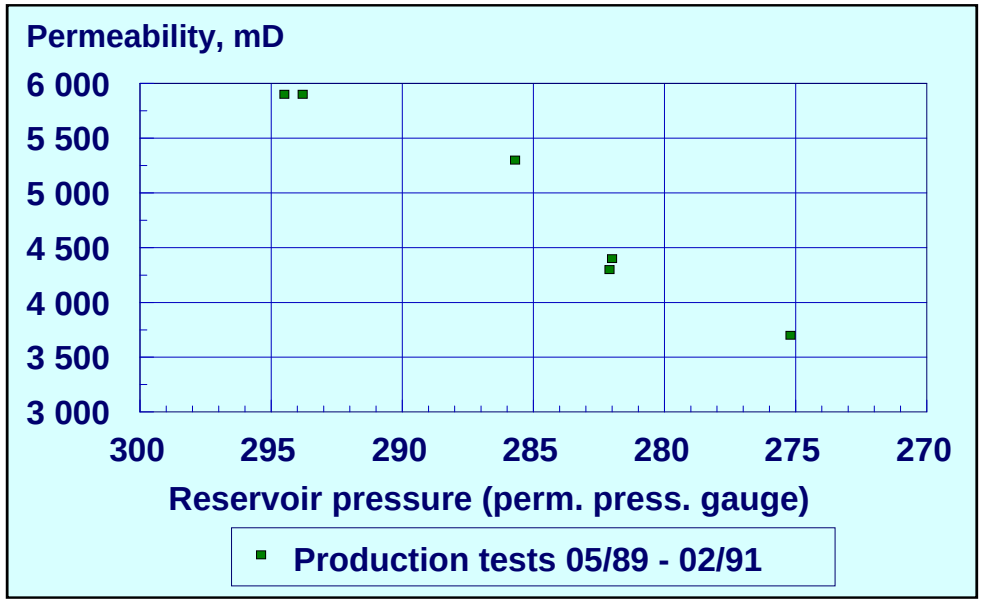



Taken from a simulation on a Valhall segment

# Examples Permeability vs. Load, Gullfaks



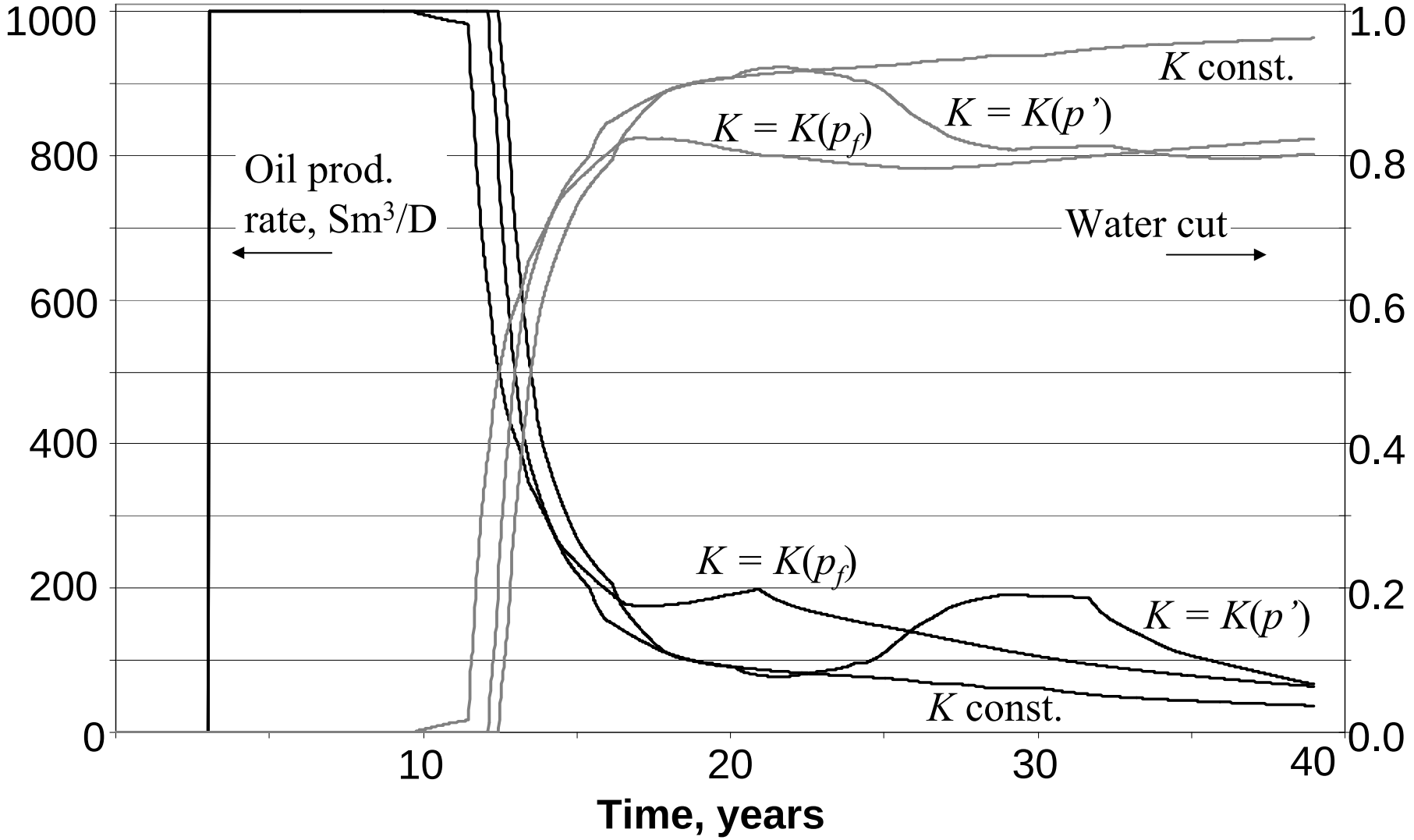



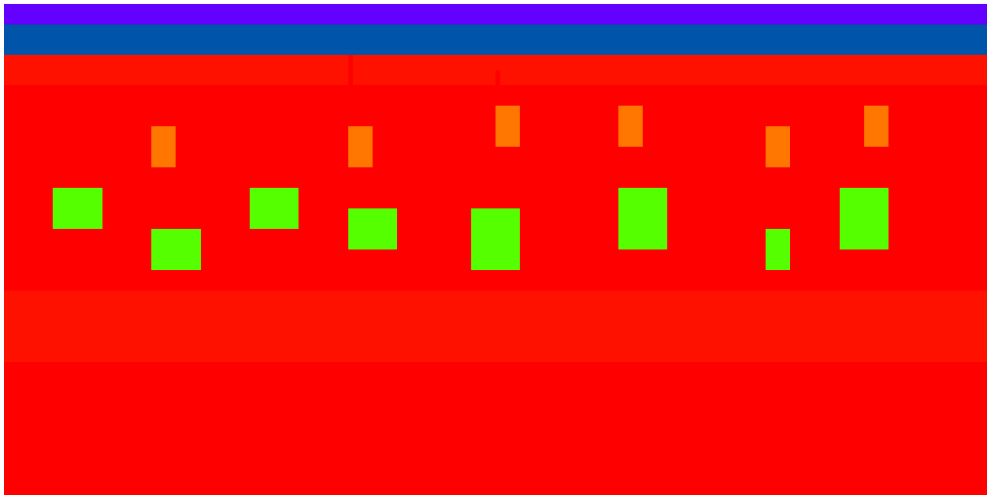

For weak sandstone / sand 90% permeability reduction at 100 bars load is not untypical.

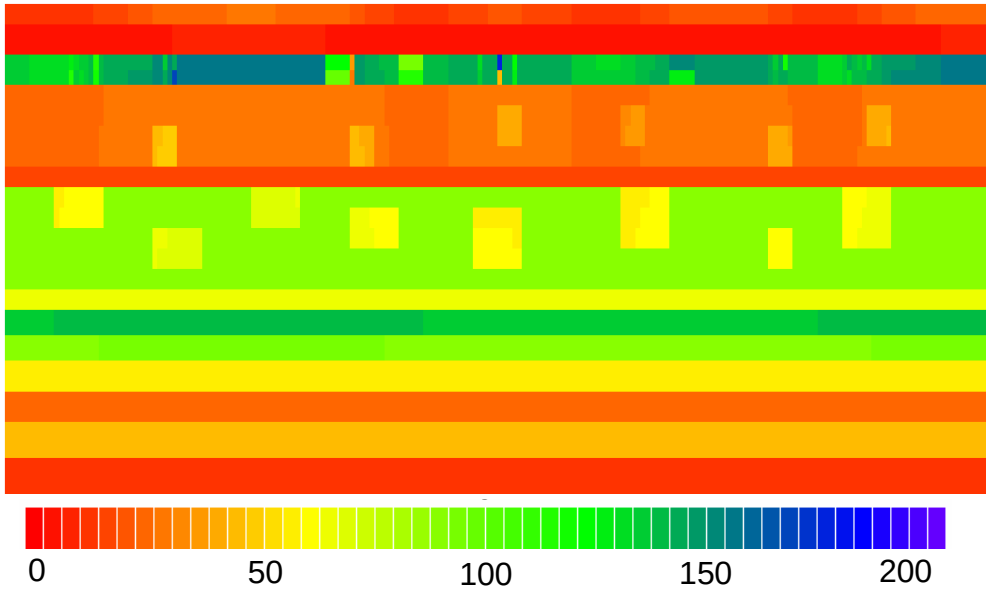
I.e.: Significant effect

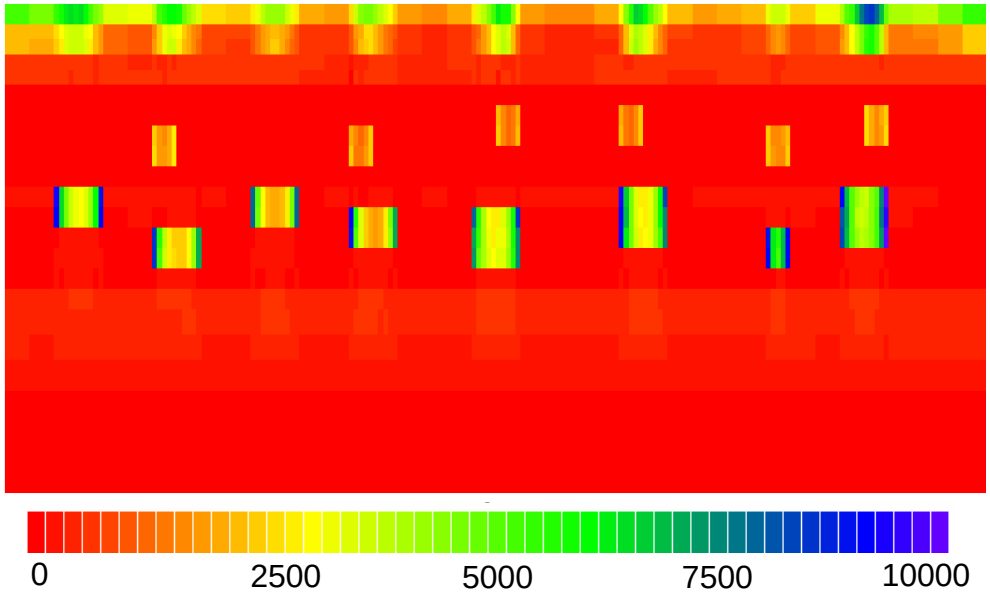
# Comparison:
# Simulated Well rates for different *K*-models



# Motivation: Comparison of permeability models



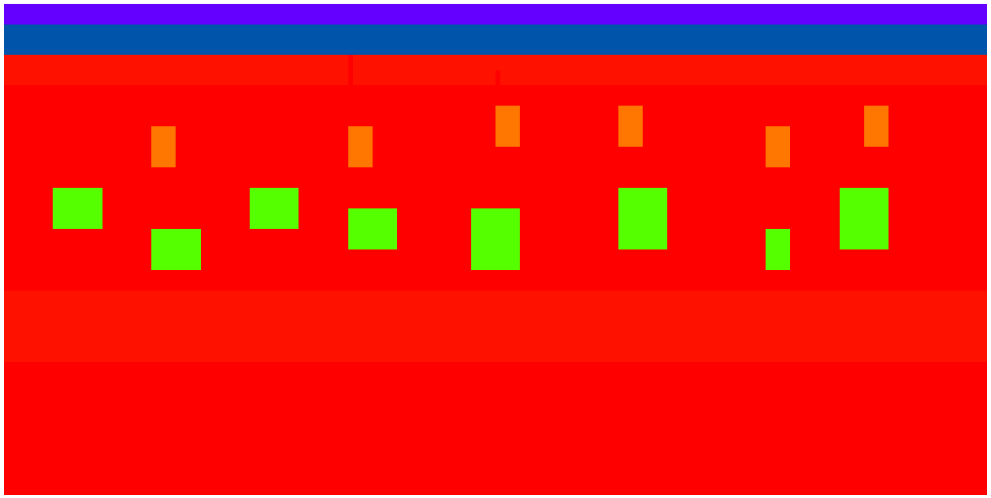



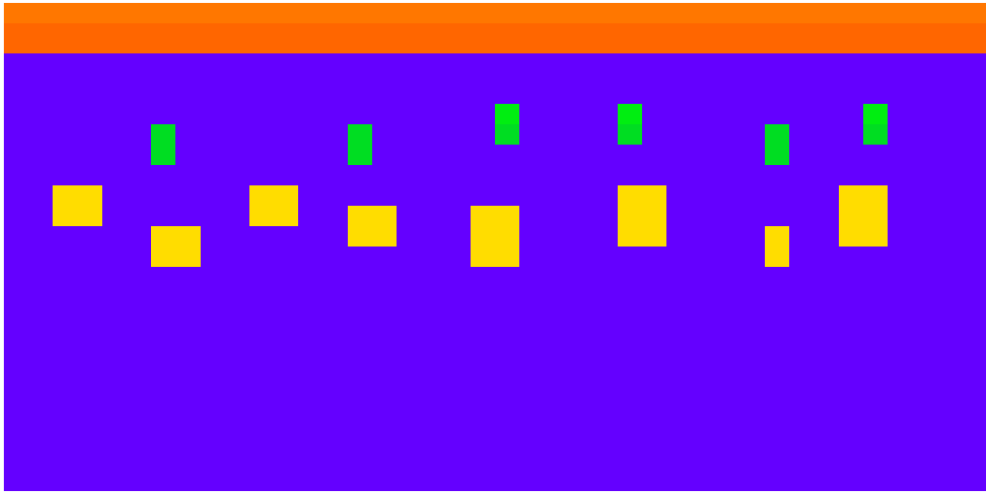



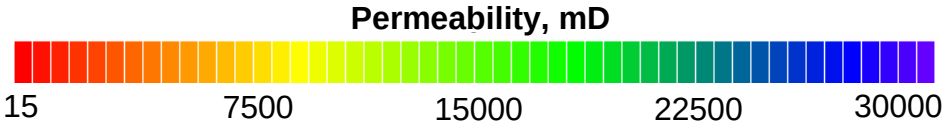

Idealized GF-model w. high-perm channels. YZ-Section 56 at restart step 34.

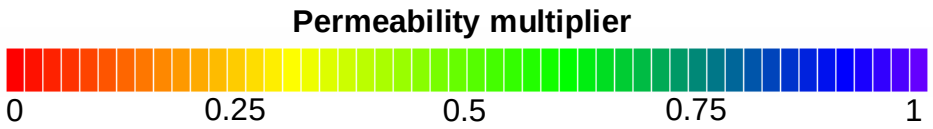
Top left: $k$ independent of load
Top right: $k = k(p_f)$ (note scale!)
Btm right: $k = k(p')$ (”Correct”)

# As prev. slide, permeability *multipliers*





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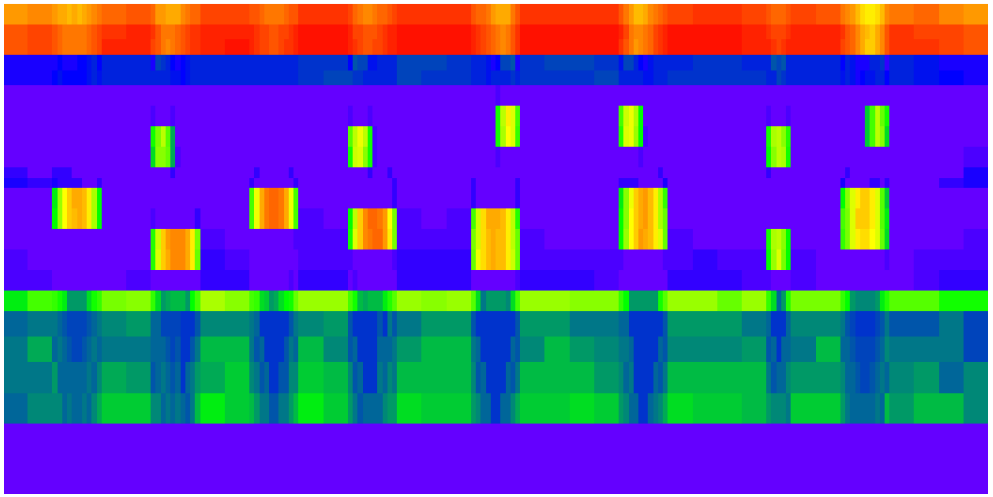


# The coupling term

Define a function $m$, called the *pore volume multiplier*:

$$m = \frac{PV}{PV_0}$$

$PV$ is pore volume, and subscript 0 denotes *initial*.

In a simulation context, $PV$ would typically be taken as cell values, while in general we could use some REV or unit reference volume.

$m$ is used in preference to porosity, but plays the same role.

Returning to the flow equations, we tacitly assumed

$$m = m(p_f) \quad (\text{or } m = m(p_f, x) \quad \text{or} \quad \phi = \phi(p_f, x))$$

$$k = k(p_f) \quad (\text{or } k = k_0 \quad \text{or} \quad k = k(p_f, x))$$

# The coupling term

The correct formulation should be:

$$m = m(p_f, p', x)$$

$$k = k(p_f, p', x)$$

which imposes a rather strong coupling between the flow and rock mechanics equations.

(Note: As $m$ and $k$ are both non-decreasing with $p'$, it is possible to define $k = k(m)$. We will therefore focus only on $m$ from now on.)

System of interest:

$$\mathbf{F}(p_f, S_w) = \mathbf{0}$$

$$\mathbf{R}(\boldsymbol{\varepsilon}, \boldsymbol{\sigma}, \mathbf{u}) = \mathbf{0}$$

$$m = m(p_f, p', \mathbf{x})$$

# Pore Volume Multiplier

Different strategies for computing the pore volume multiplier from effective stress, or strain, have been proposed.
The most popular (?) was proposed by Thomas, Chin, and Sylte *et al.*:

During compression the porosity is changed due to grain reorganization.
In addition, the reference bulk volume is reduced:

$$\text{New effective porosity}: \ \phi = 1 - (1 - \phi_0) e^{\Delta \varepsilon_p}$$

$$\text{Updated PV - mult}: \qquad m = \frac{e}{e_0} = \frac{\phi/(1-\phi)}{\phi_0/(1-\phi_0)}$$

(subscript 0: initial value.
$e$: voids volume)

This is the model used on Valhall

# The coupling term

Ideally the full system (*) should be solved at each time step. As seen next, a fully implicit scheme is needed.

$$\left.\begin{aligned} \mathbf{F}(p_f, S_w) &= \mathbf{0} \\ \mathbf{R}(\boldsymbol{\varepsilon}, \boldsymbol{\sigma}, \mathbf{u}) &= \mathbf{0} \\ m &= m(p_f, p', \mathbf{x}) \end{aligned}\right\} \quad (*)$$

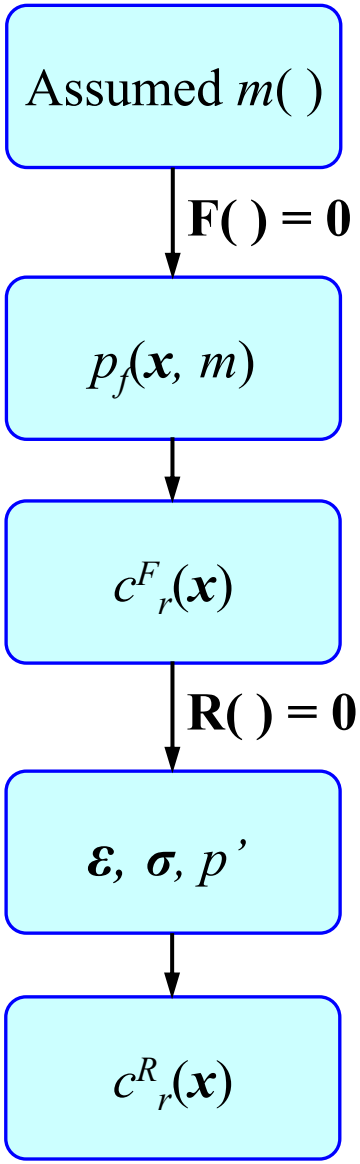
Assume (*) is solved by a staggered (implicit) scheme.
I.e. first $\mathbf{F}^{t+\Delta t}(\,) = \mathbf{0}$ is solved, and the found $p_f(\mathbf{x}, t+\Delta t)$ used in $\mathbf{R}^{t+\Delta t}(\,) = \mathbf{0}$ (for initializing of effective stress).

However, $m(\,)$ is dependent on $p'$, and some "guestimate" has to be done prior to solving $\mathbf{F}^{t+\Delta t}(\,) = \mathbf{0}$. The natural choice is the $m(p_f)$ assumed for the stand-alone problem.

Note that the computed $p_f$ is strongly dependent on the chosen $m(\,)$, as the degree of rock compression has strong influence on fluid pressure.
To find the "correct" $m(\,)$ the rock mech problem must be solved...

# Flow chart for computing compaction in $x$

$$\left.\begin{aligned}&\mathbf{F}(p_f, S_w) = \mathbf{0}\\&\mathbf{R}(\boldsymbol{\varepsilon}, \boldsymbol{\sigma}, \mathbf{u}) = \mathbf{0}\\&m = m(p_f, p', \mathbf{x})\end{aligned}\right\} \quad (*)$$



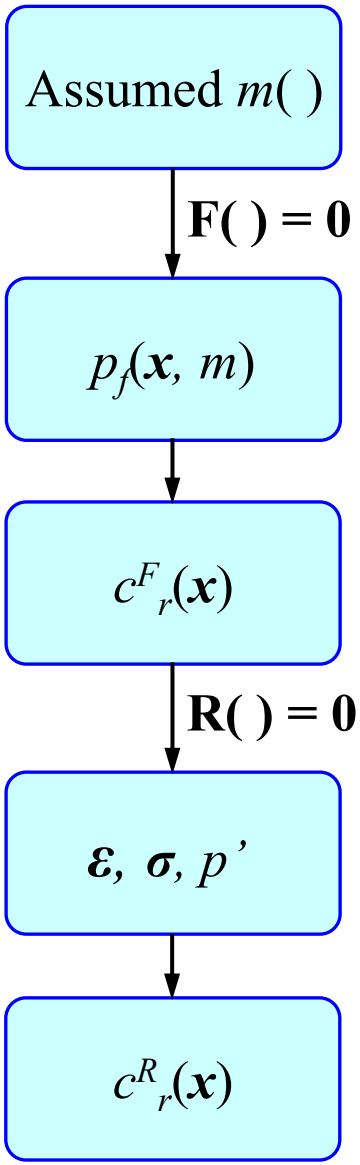

Pressure based on assumed $m(\,)$.

Compaction in $\boldsymbol{x}$ from flow equations

Rock mech parameters based on total energy from $c^F{}_r$

Compaction in $\boldsymbol{x}$ from stress (strain)

# Flow chart for computing compaction in $x$




Probably wrong

$\rightarrow$ wrong

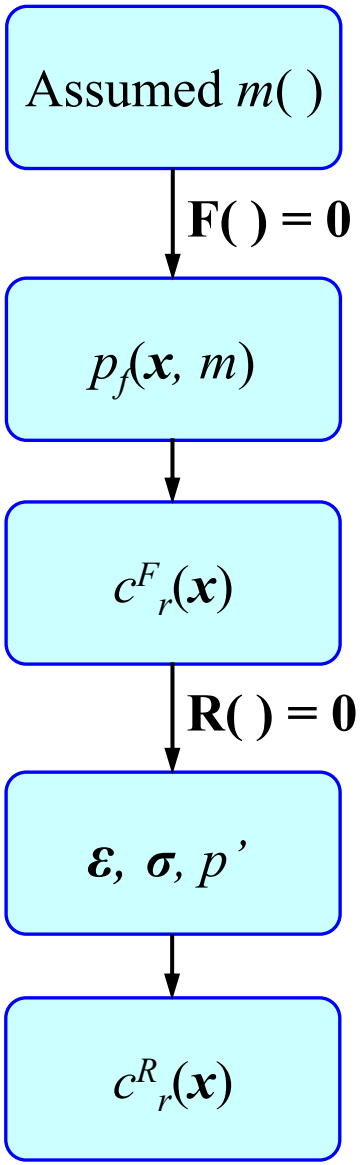
$\rightarrow$ wrong

$\rightarrow$ wrong (as total energy from $c^F{}_r$ was wrong)

$\neq c^F{}_r$ unless we're extremely lucky

$$\left.\begin{aligned} &\mathbf{F}(p_f, S_w) = \mathbf{0} \\ &\mathbf{R}(\boldsymbol{\varepsilon}, \boldsymbol{\sigma}, \mathbf{u}) = \mathbf{0} \\ &m = m(p_f, p', \mathbf{x}) \end{aligned}\right\} \quad (*)$$

# Flow chart for computing compaction in $x$




We can't expect compaction from the flow equations to equal compaction from the RM equations by this scheme.

$$\left.\begin{aligned} &\mathbf{F}(p_f, S_w) = \mathbf{0} \\ &\mathbf{R}(\boldsymbol{\varepsilon}, \boldsymbol{\sigma}, \mathbf{u}) = \mathbf{0} \\ &m = m(p_f, p', \mathbf{x}) \end{aligned}\right\} \quad (*)$$

$c^F{}_r$ was a "wild shot", but $c^R{}_r$ was based on it. So none of them are reliable.

However, a good starting point for an iteration scheme:
After computing $c^R{}_r(\boldsymbol{x})$, compute ratio

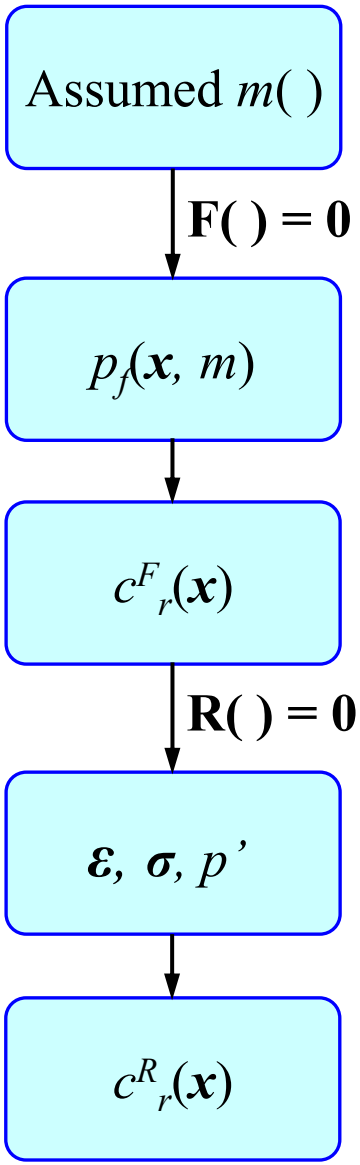
$$r_c(\mathbf{x}_i) = \frac{c_r^R(\mathbf{x}_i, t+\Delta t)}{c_r^F(\mathbf{x}_i, t+\Delta t)}$$

Set $c^F{}_r(\boldsymbol{x}_i\ ,\ t)$ *= $r_c(\boldsymbol{x}_i)$ for all $\boldsymbol{x}_i$, and repeat calculations until $|c^F{}_r(\boldsymbol{x}_i) - c^R{}_r(\boldsymbol{x}_i)| < tol.$ everywhere.

Which more or less "proves" that the scheme has to be fully implicit for the coupled system.

Problem: This is going to take time.
Typically, one RM-solve takes 10-100 times one F-solve...

# Flow chart for computing compaction in $x$

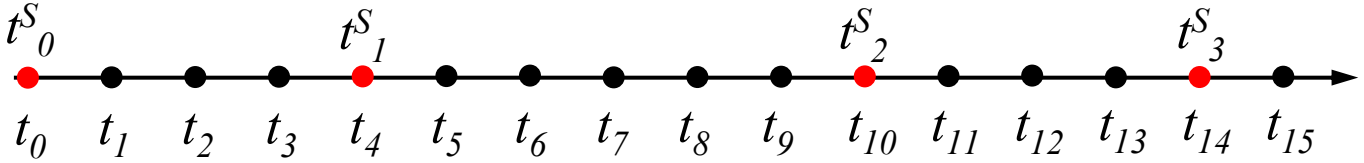



$$\left.\begin{aligned}\mathbf{F}(p_f, S_w) &= \mathbf{0}\\ \mathbf{R}(\boldsymbol{\varepsilon}, \boldsymbol{\sigma}, \mathbf{u}) &= \mathbf{0}\\ m &= m(p_f, p', \mathbf{x})\end{aligned}\right\} \quad (*)$$

Modification:

**F**( ) is solved at all time steps.

**R**( ) is only solved at selected steps: stress steps




Mechanism is the same as before, but we solve fewer rock mech systems.

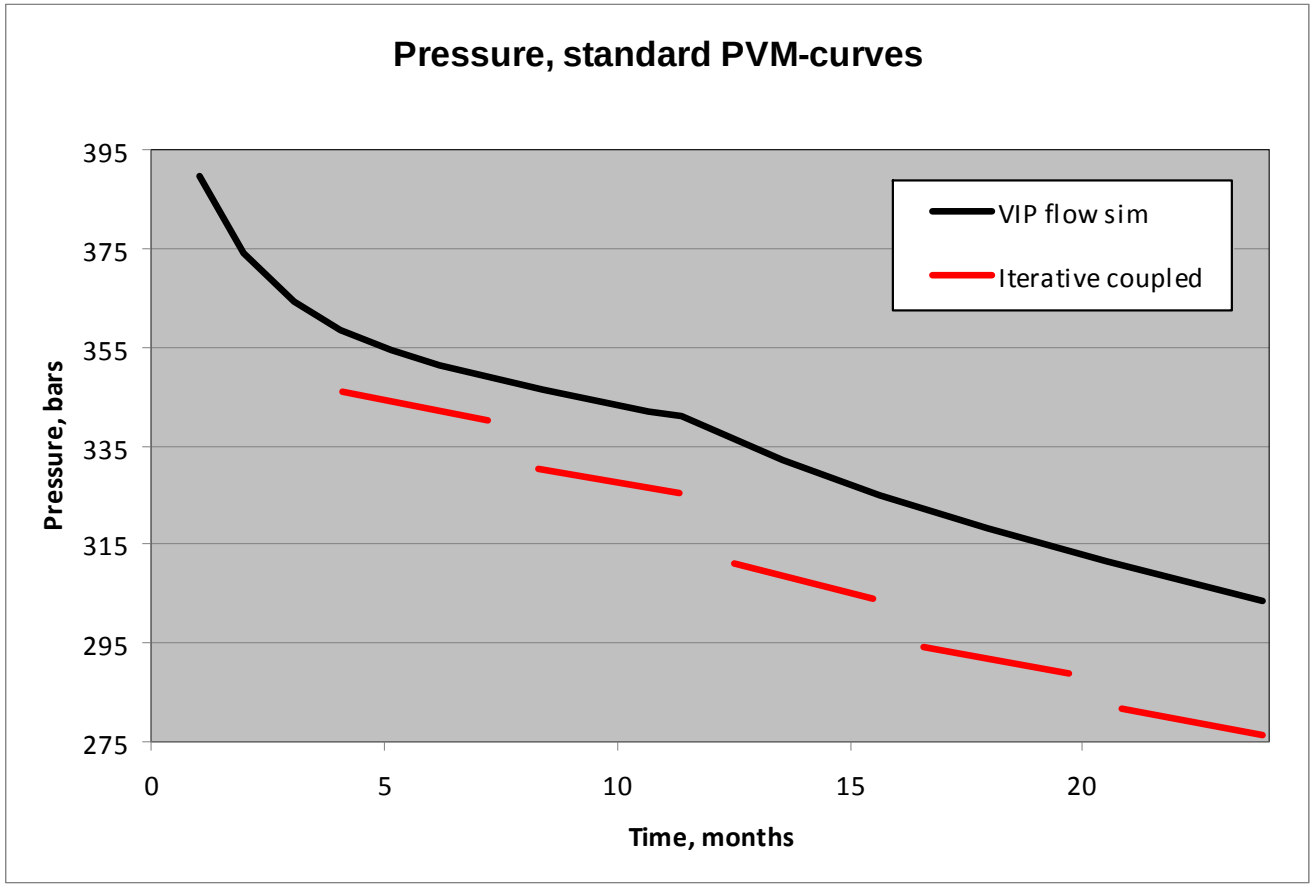
On the other hand, the difference between the computed compactions will probably be larger (more iterations?)

Also, the fluid pressures will only be correct at the stress steps, and ”drift away” in between.

# Options in coupled simulation

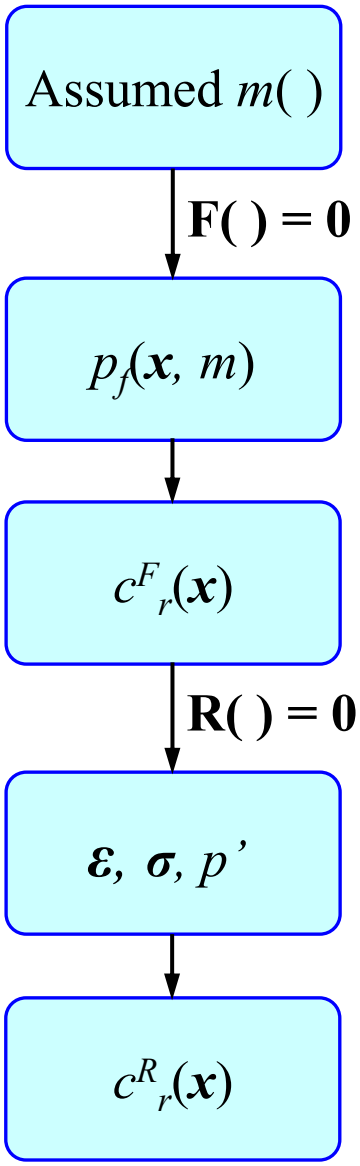
1. **Explicit coupling**: Flow simulator run between stress steps
   Rock mech simulator run at stress steps. No feedback
   (solution accepted as is)
   - Cannot expect correct compaction computations
   - Hence also not fluid pressure
   - Fastest alternative
2. **Implicit coupling**: (Coupling with pore volume iterations)
   As explicit, except the RM-computed compaction is fed back into the flow simulator, and the cycle Flow-sim – RM-sim – feedback is repeated until convergence (or some max #loops)
   - Accurate compaction & fluid pressure at stress steps
   - Not-so-accurate in between stress steps
   - #iters = 1: ”Two-way coupling” or ”expl. coupling w. feedback”
   - Relatively expensive
3. **Fully implicit solution** of coupled system at each time step
   - accurate, but in general very expensive
   - lack of fully coupled simulators which offer all desired options

# ”Not-so-accurate between stress steps”




Industry standard approach, using coupled simulation with pore volume iterations. As seen, the pressure (and compaction) is correct *only* at the stress steps, and follows the input curve trend beween these.

# Flow chart for computing compaction in $x$

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$$\left.\begin{aligned} &\mathbf{F}(p_f, S_w) = \mathbf{0} \\ &\mathbf{R}(\boldsymbol{\varepsilon}, \boldsymbol{\sigma}, \mathbf{u}) = \mathbf{0} \\ &m = m(p_f, p', \mathbf{x}) \end{aligned}\right\} \quad (*)$$

Wouldn't it be wonderful if this guy was correct in the first place?

# Construction of an optimal $m(p_f\,;\,p')$

The goal is to construct an *m*-function for use when solving the flow equations, such that the resulting pressure state is an optimal initializer for solving the rock mechanics equations.

I.e., when using this ”pseudo” *m*-function in the flow simulator, an accurate compaction state will ideally be available already at the flow sim stage.

Should the flow-sim compaction not be of acceptable quality, it should at least be the best possible starting point (initializer) for solving the rock mechanics equations.

# Construction of an optimal $m(p_f ; p')$

Neglecting grain compression, the effective stress was,

$p' = p - p_f$.

Here,

$p = p(\boldsymbol{x}, t)$:

primarily governed by external forces, which don't change much during reservoir depletion. $p$ can have, and often has, large spatial variation, but not with time for fixed $\boldsymbol{x}$ (subsidence, arching).

$p' = p'(\boldsymbol{x}, t)$:

primarily governed by pore collapse / grain reorganization. May change rapidly both in space and time.

Hence we can typically expect: (for primary loading)

$$\left.\frac{\partial p}{\partial p_f}\right|_x << \left.\frac{\partial p'}{\partial p_f}\right|_x \qquad (**)$$

# Construction of an optimal $m(p_f\,;\,p')$

We can always split $p'$ as:

$$p'(p_f, p; \mathbf{x}) = p_p'(p_f; \mathbf{x}) + \tilde{p}'(p_f, p; \mathbf{x})$$

where $p_p'$ is the projection of $p'$ on the $p_f$-plane.

Taking (**) for valid, then:

$$p'(p_f, p; \mathbf{x}) \approx p_p'(p_f; \mathbf{x})$$

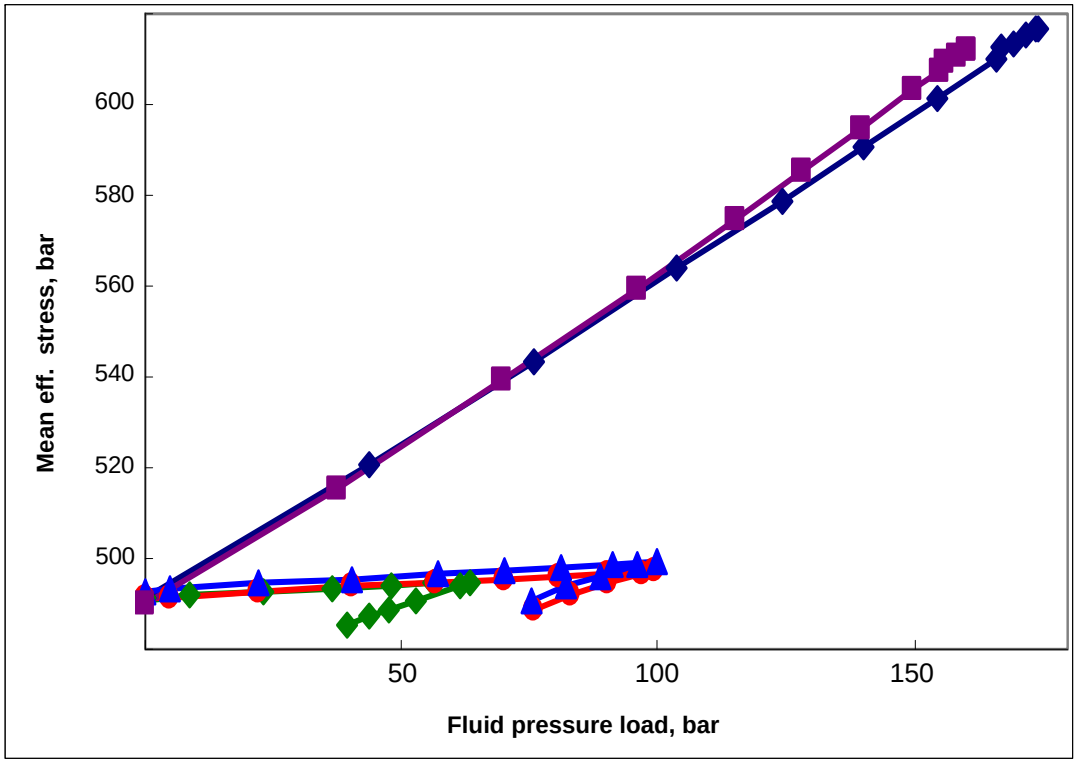
I.e., During primary load, *locally* (for fixed $\boldsymbol{x}$) we can expect $p'$ to be a function of fluid pressure alone.
**(Key observation 1)**

# Construction of an optimal $m(p_f ; p')$

In numerous numerical experiments,
the correlation between mean effective stress
and fluid pressure has been recorded
for fixed $\boldsymbol{x}$ (i.e. in a single cell)

Results consistently agree
with key observation 1.

Figure shows some examples
from a Valhall study.



# Construction of an optimal $m(p_f ; p')$

Key observation 2:
The rock mechanics equations are energy-conserving.

I.e.: Total (compaction) energy in the system is determined by the reservoir state as delivered from flow equations.

As long as only the scaling of *m*-functions is changed beween cases,
i) The fluid pressure *level* changes, but the qualitative *distribution* is the same
ii) The total stress is determined by external forces.
Effective stress is governed by the fluid pressure.
Hence, scaling the *m*-functions influences the effective stress level, but not distribution.
(→ Same goes for compaction)

# Construction of an optimal $m(p_f\,;\,p')$

A consequence of key observations 1 & 2 is that we can treat $m$ as we did with $p'$:

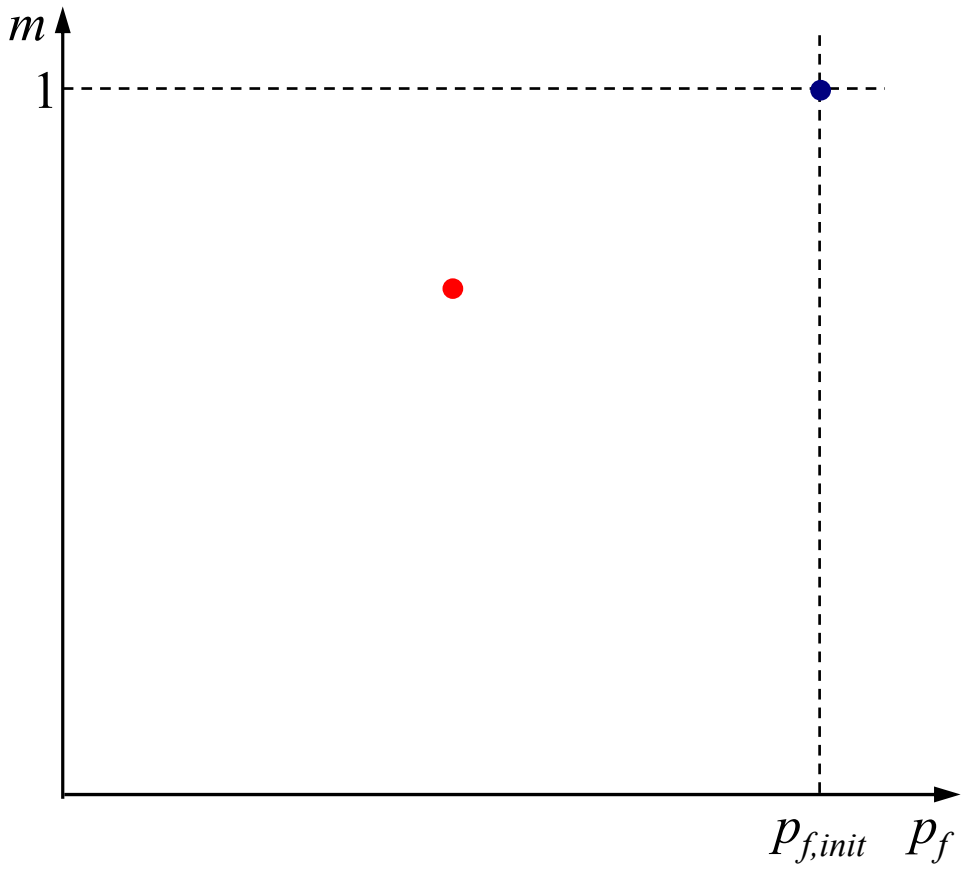
$$m(p_f, p, \mathbf{x}) = m(p_f, \mathbf{x}) + \tilde{m}(p_f, p, \mathbf{x}) \qquad (***)$$

with typically

$$m(p_f, p, \mathbf{x}) \approx m(p_f, \mathbf{x})$$

Note: This is never going to be wrong anyway, as (***) will be used in the rock mechanics part of the coupling. If the approximation is invalid / bad for some $\boldsymbol{x}$, this will be corrected by the procedure.

# Construction process

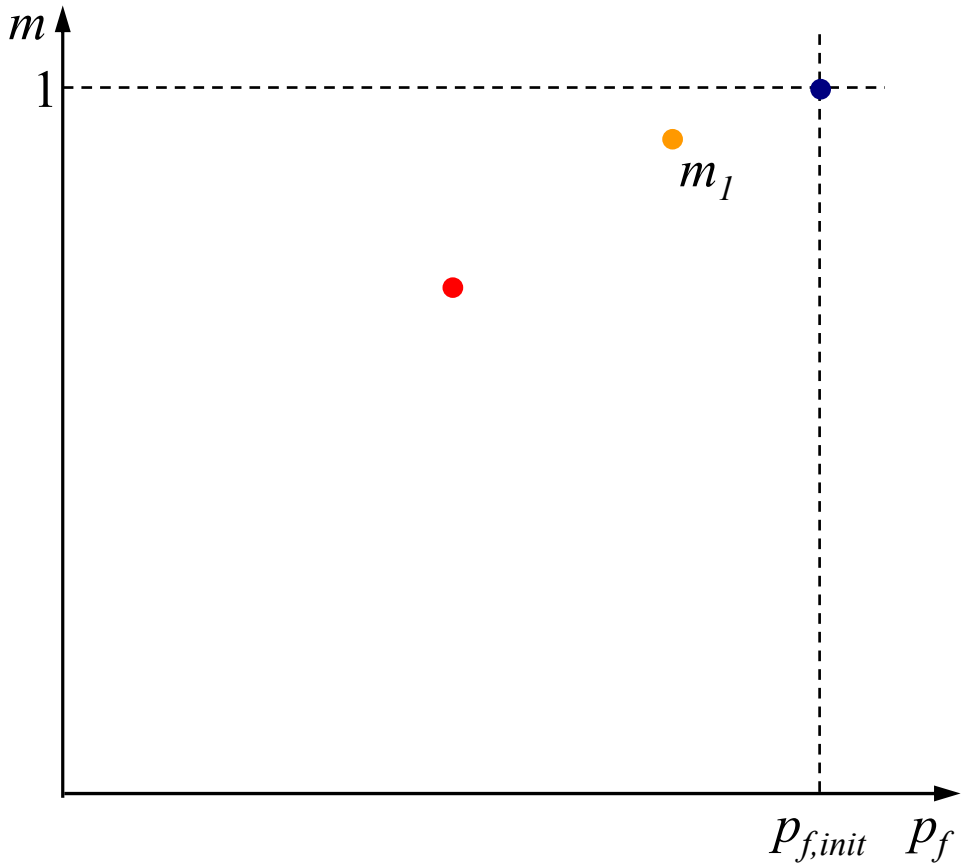



Keep ***x*** fixed, i.e. in a simulation environment we study one single cell (element).

At initial pressure the pore volume is at unloaded value ($m = 1$) (blue)

At time *t,* the true compaction, and thereby correct pressure in the cell is unknown but shown on figure (red point).

# Construction process




Using an assumed $m(p_f)$, solve **F( )=0** at time $t_1$.

Solve **R( ) = 0** with the reservoir state from **F( ) = 0** as initializer.

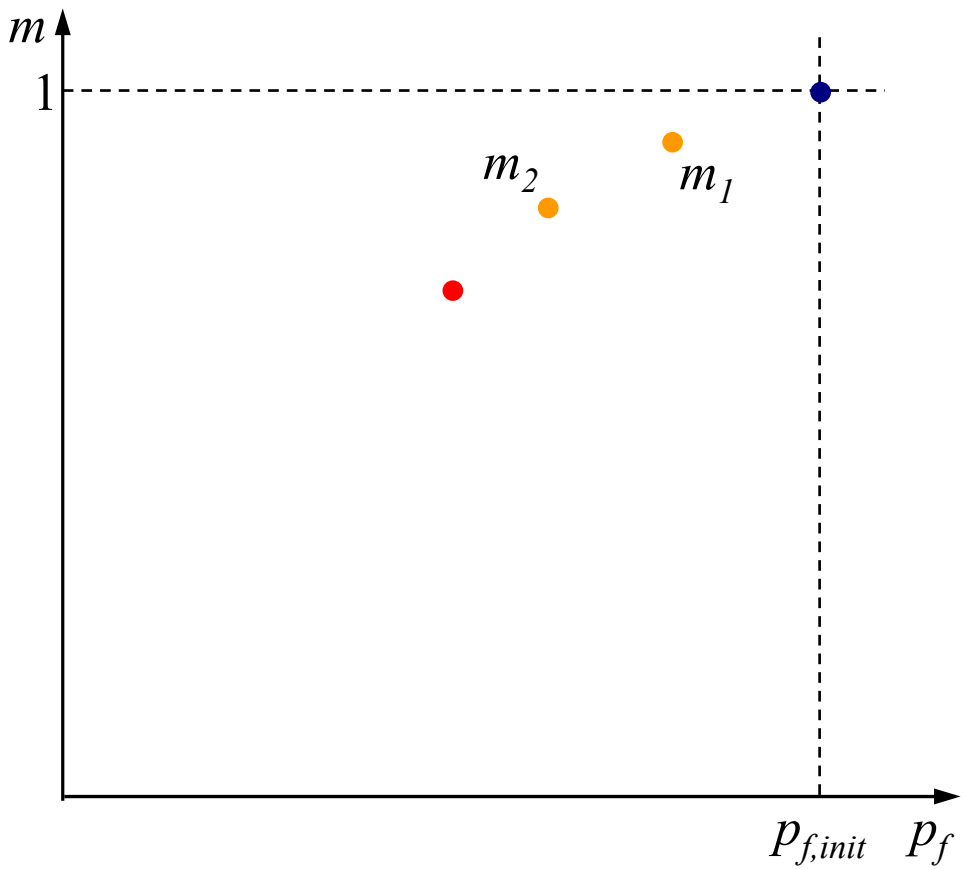
Compute cell-value $m$ from strain $\varepsilon_p$.

Record $p_{f,1}$ and $m_1$.
( = $p_f(t_1)$ and $m(p_{f,1})$. )

Repeat for times *t2, t3, t4, ...*

Record the $(p_i, m_i)$, $i$ = 2, 3, 4, ...

# Construction process

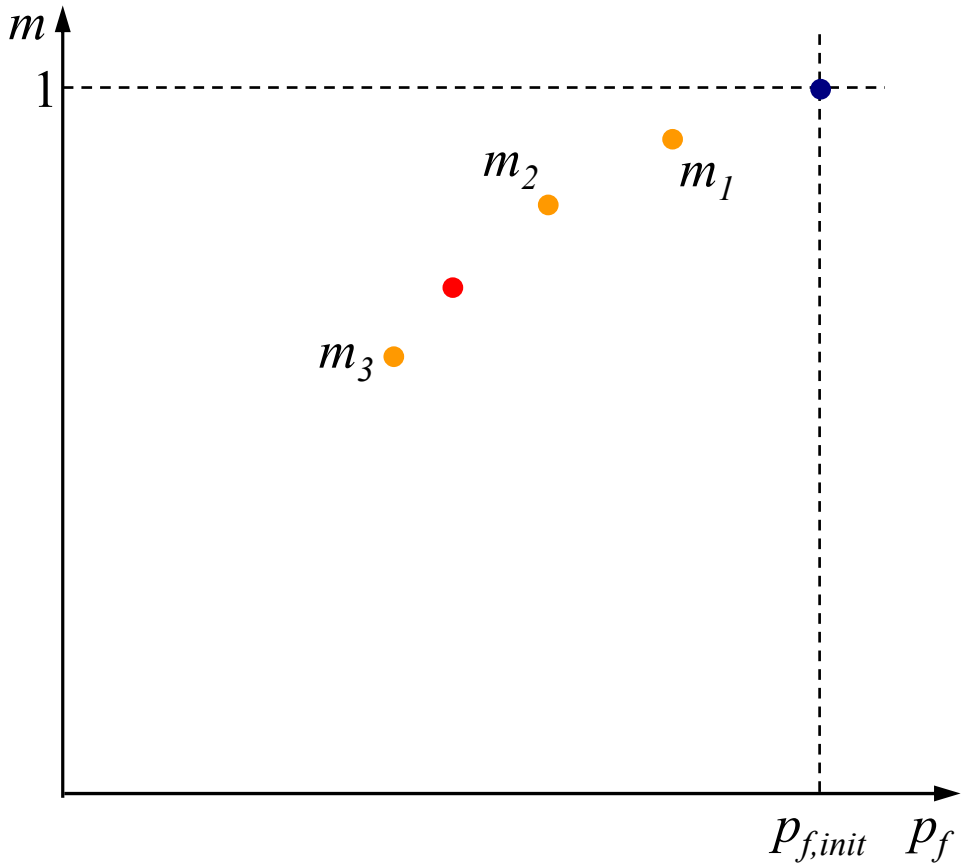



Note that all points ($p_i$, $m_i$) are *permitted solutions* of the system (*), with the production and material data provided, and hence represents a possible reservoir state.
As $p' = p'(p_f)$, the $m(p_f)$ is unique: the only point in the ($p_f$, $p$)-plane honouring the reservoir state.

The set of points ($p_i$, $m_i$) hence define a unique curve of permitted states in this cell, for the reservoir description in question.

# Construction process

Repeating for times $t_3$, $t_4$, ...



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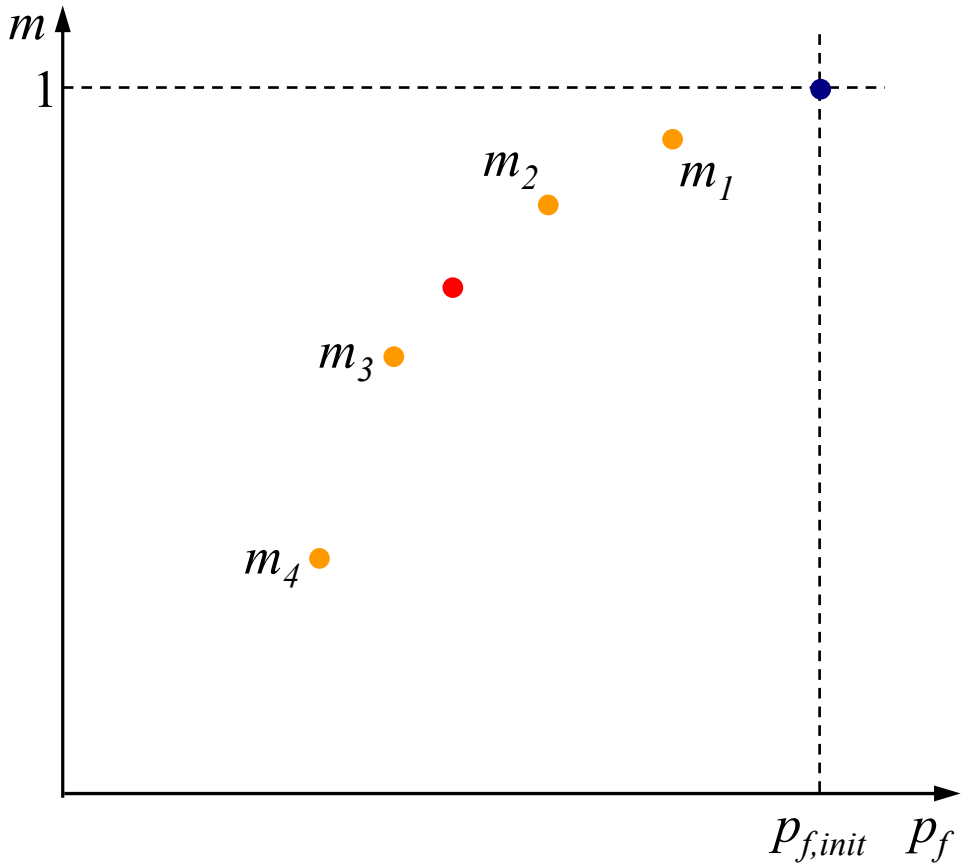
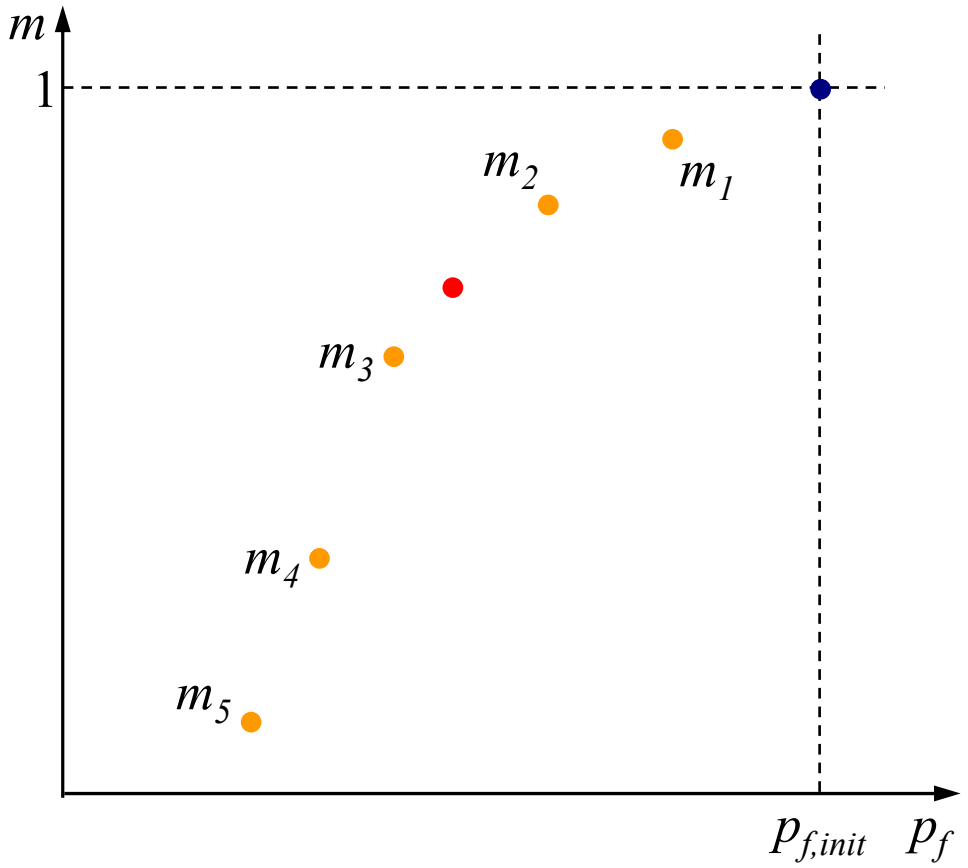
Repeating for times $t_3$, $t_4$, ...

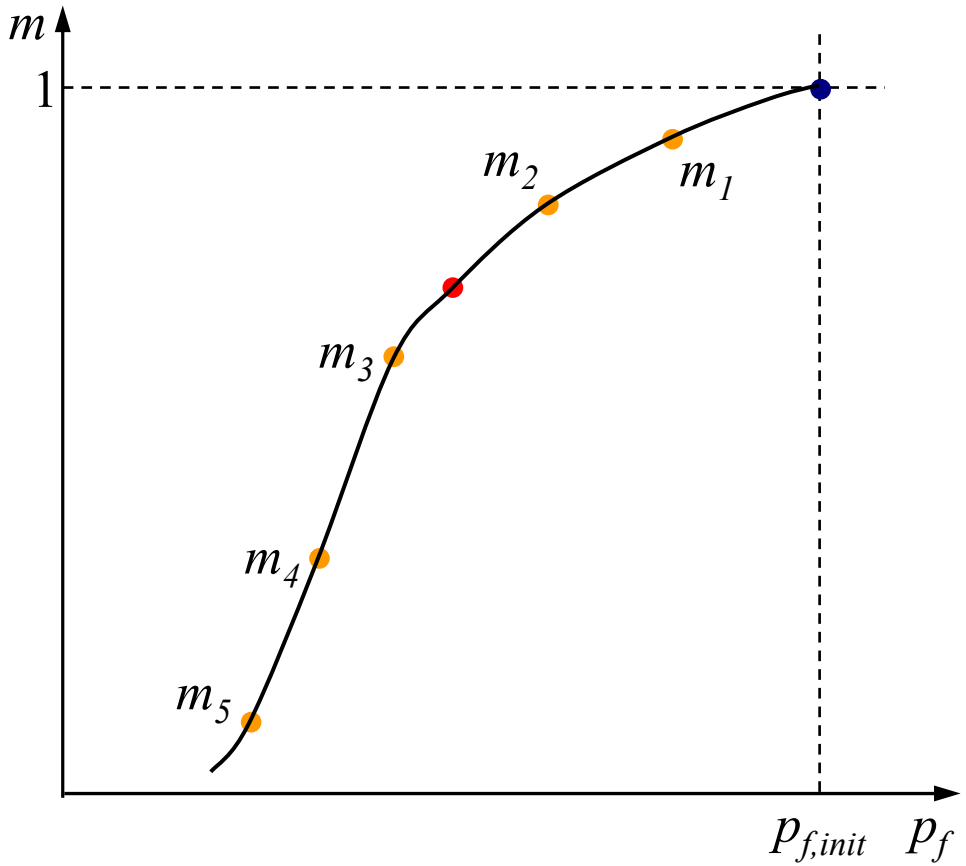


# Construction process

Repeating for times $t_3$, $t_4$, ...



# Construction process

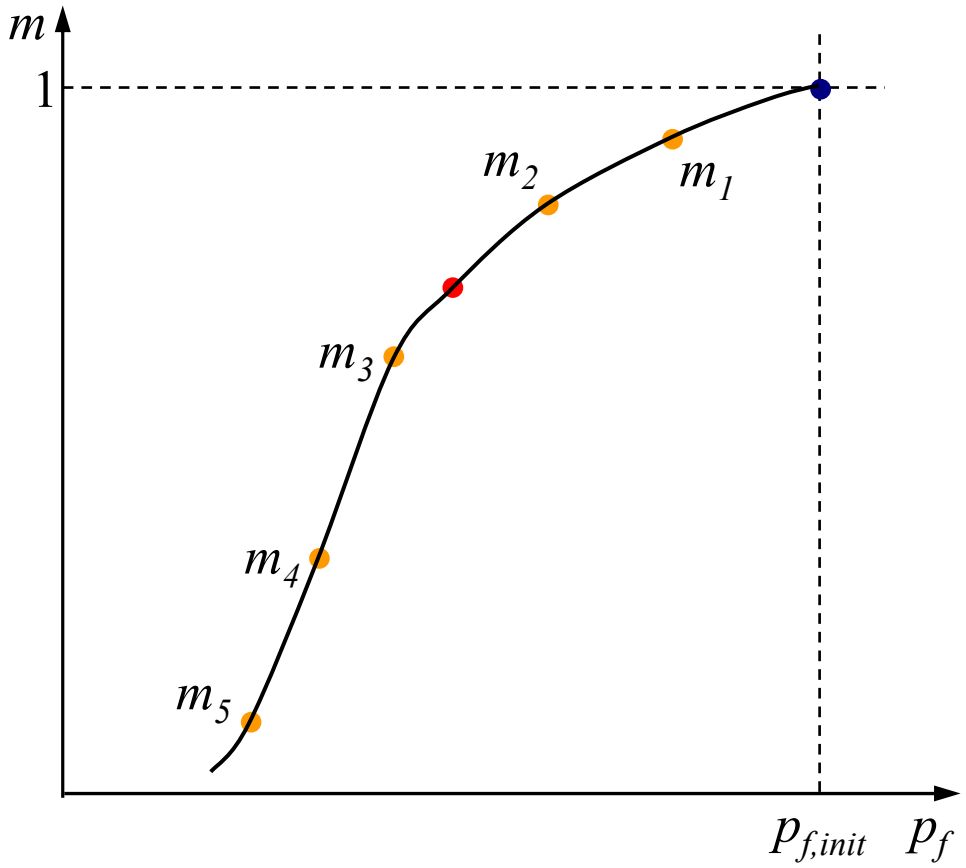



By the construction process, the (unknown) true reservoir state must lie on this curve.

Hence if this curve is used when solving **F( ) = 0**, the correct values of fluid pressure and PV-mult will be computed.

As a consequence, the stress *level* will be correctly computed from **R( ) = 0**, but for the purpose of computing compaction we no longer need to solve this system.

# Construction process



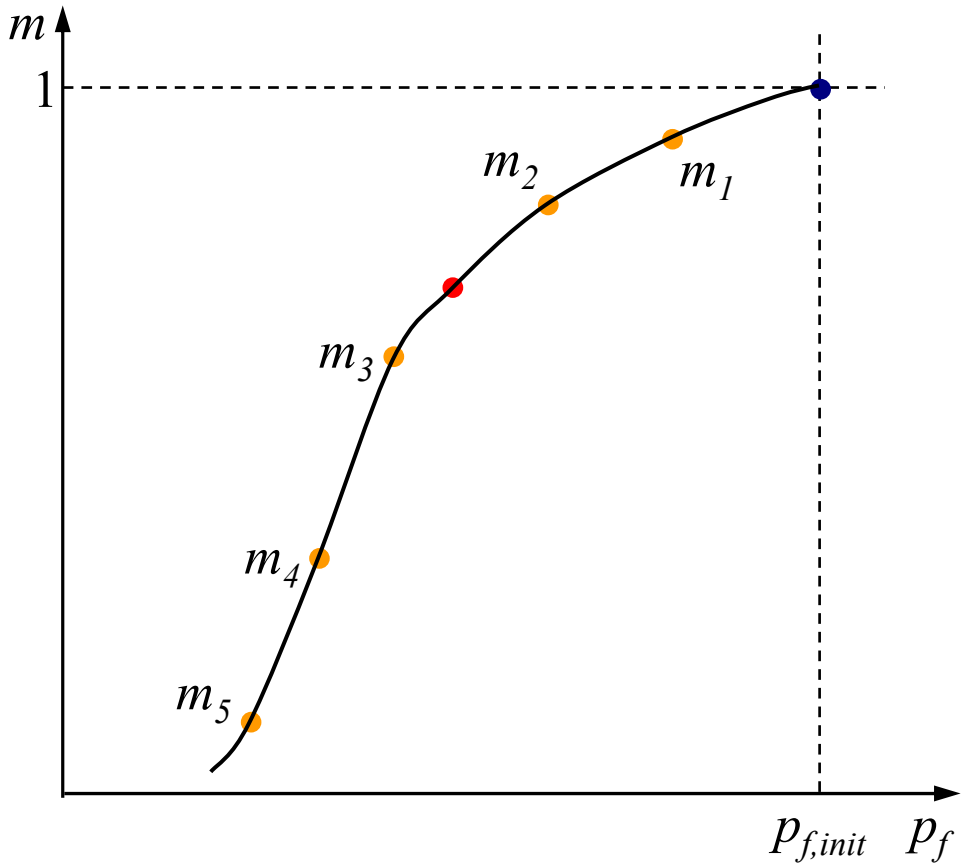

On the figure, the points $m_1$, $m_2$, ... are in decreasing order.

This is no coincidence:

All the points used to determine $m(p_f)$ must be on the normal compression line (primary compression).
Any unloading-reloading points must be removed from the procedure.

Other consistency requirements must also be honoured.

# Construction process




Next (or obviously simultaneously) we repeat this procedure for all cells in the mesh, ending up with one *m*-curve for each cell.

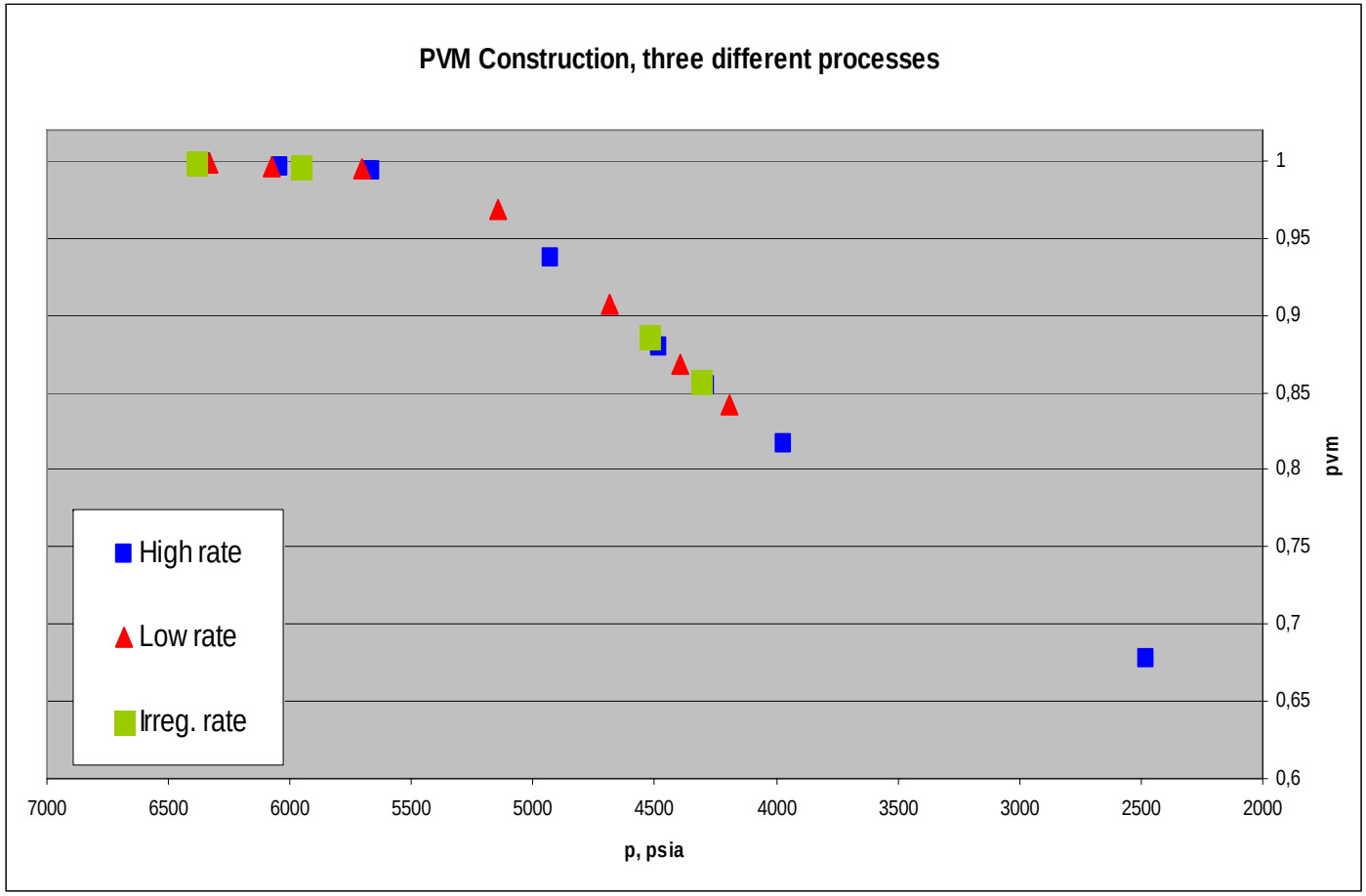
While it's possible to use this set of curves when solving **F( ) = 0**, for convenience we group "equal" curves into common material regions.

(For meshes with more than about 800K cells this also becomes necessary due to CPU memory limits.)

# PVM curve Construction is Independent of Process

By the theory, the constructed *m*-curve should only depend on the reservoir description, not on the dynamics of the production.

The figure shows three different dynamic scenarios, which all generated the same *m*-curve. (”Exp. proof” of statement.)



# Comments

By the theory, the procedure works if the reservoir state is the same when the different *m*-values are generated.

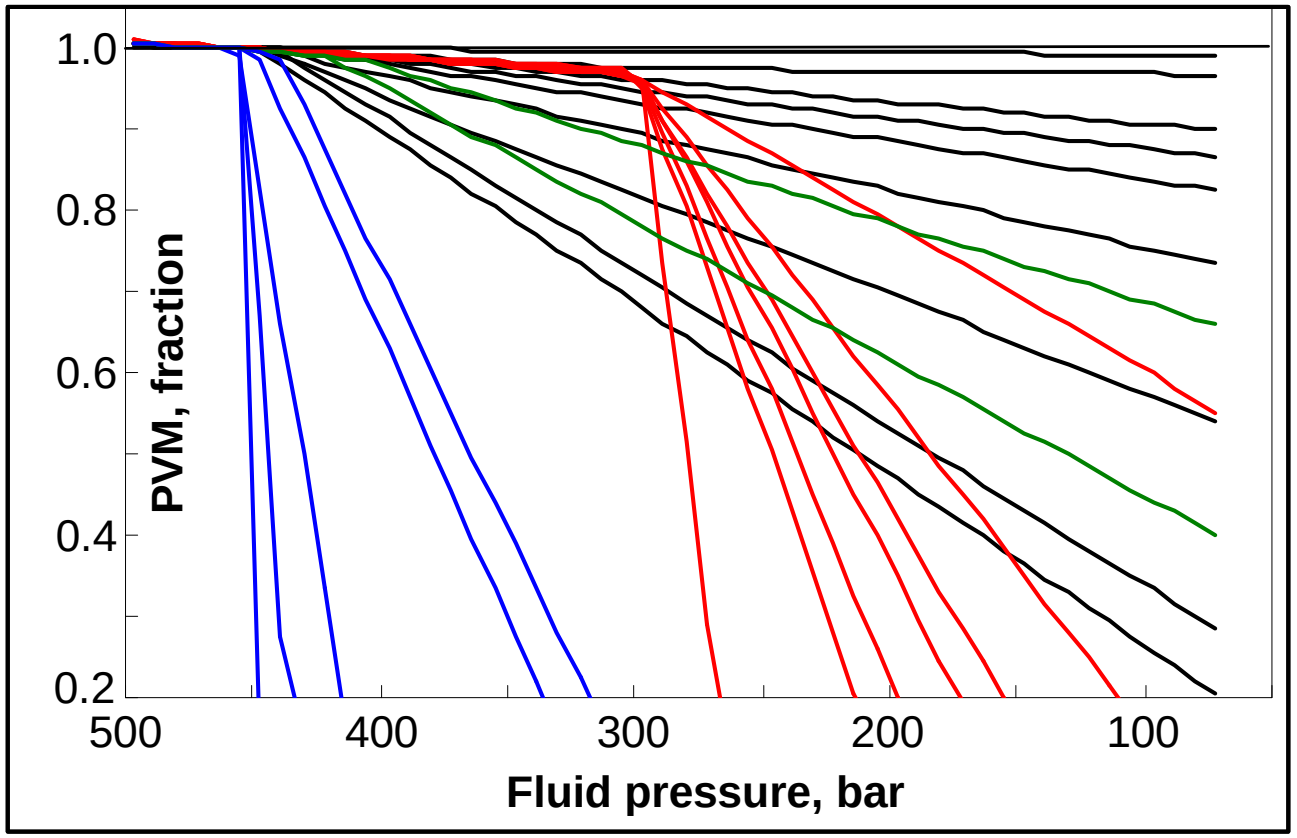
I.e. material description, reservoir geometry, petrophysics, boundary conditions, (e.g. well positions), ...
However, no restrictions on well rates (e.g. increased rate → higher drawdown → lower near-well pressure → larger compaction – is taken care of by pressure – stress relationship).

Change in petrophysics without changing rock mechanics properties works fine.

Changing well positions should require a redermination of *m*-curves. However, when testing this we found that the original family of *m*-curves works reasonably well also for altered well pattern.

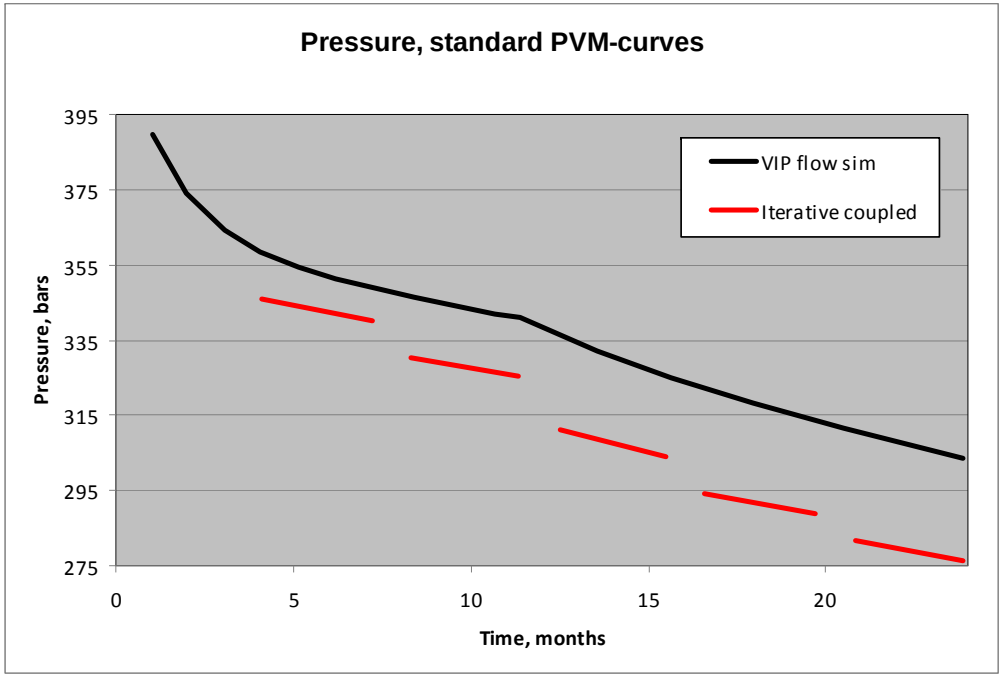
Hence, the procedure is well suited for e.g. geostatistical analysis, as only one set of stress simulations is needed prior to the batch of heterogeneity runs.

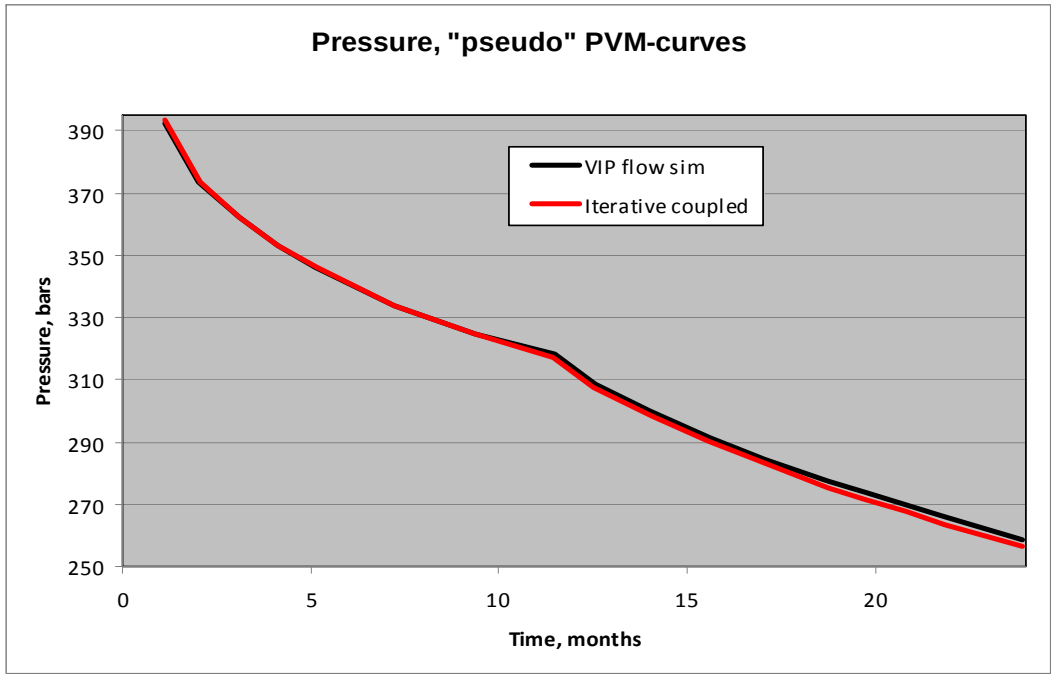
# Some results from simulations




Excerpt of generated *m*-curves from a *single material* (Valhall-study).
Clearly shows that the difference from using a single curve is *real*.

# Some results from simulations

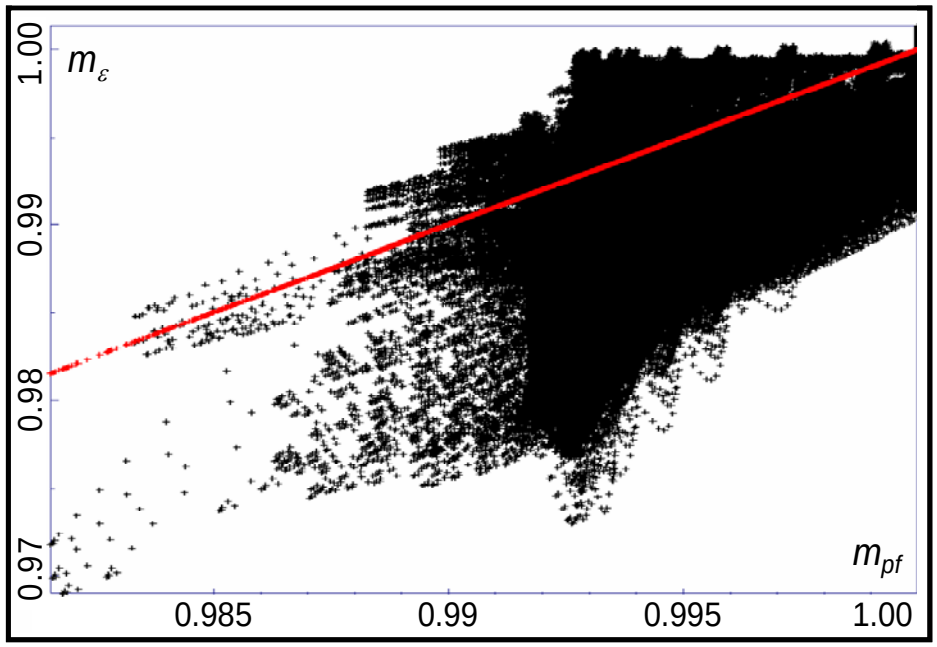


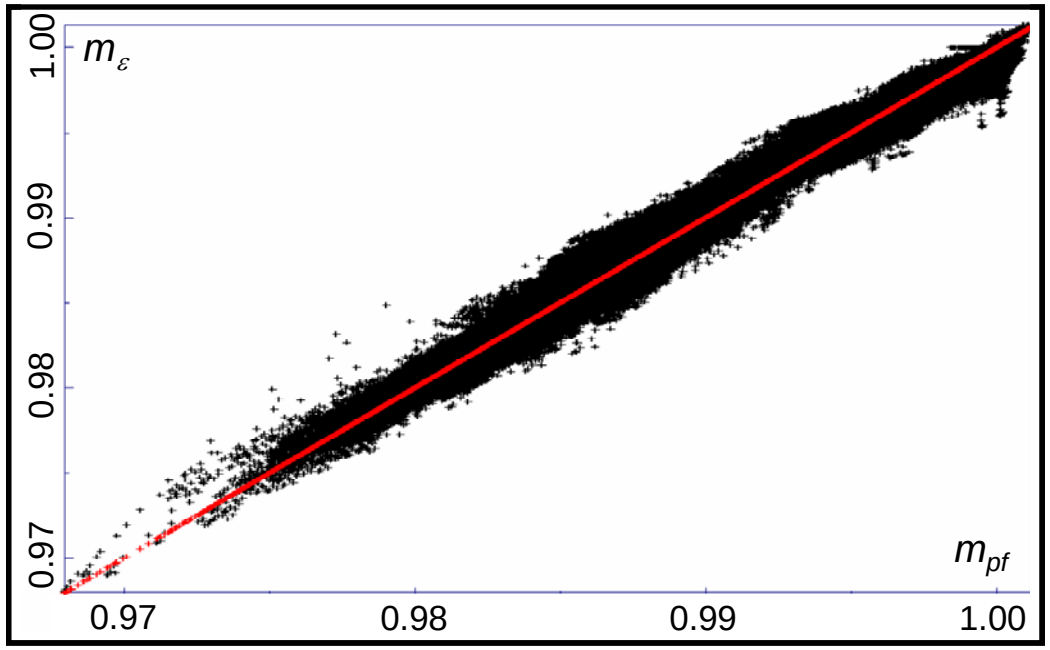




Industry standard approach, using coupled simulation with pore volume iterations. As seen, the pressure (and compaction) is correct *only* at the stress steps, and follows the input curve trend beween these.

Stand-alone flow simulation and coupled simulation with 7 pore volume iterations, both using the modified *m*-curves in the flow simulation.
As seen the two approaches generate almost identical results, the former at a fraction of the time.

# Some results from simulations




The plots show correlation:
$m(p')$ ”correct” by iterative coupled simulation
$m(p_f)$ from flow simulation

Left fig.: Using ”standard” $m$-curves. Right fig.: Using mod. $m$-curves.
(Red line: Perfect correlation)

# Implementation

The procedure has been implemented in a program, **`mech2sim.`**

This program is in regular use by the BP / Valhall project.

The plan is to start using it on the fourth generation full field model this fall.

And yes, there are lots of detail issues which I haven't mentioned.

# Conclusions

- The modified (”pseudo”) $m(p_f)$-curves
  - Can be constructed with about the same effort (CPU-time) as one explicit coupled run, normally less
  - Provides input data that allows the flow simulator to compute compaction and pressure with great accuracy (comparable to fully implicit solutions)
  - Often allows for running actual simulations with a stand-alone flow simulator
  - When coupled simulations are required, they can be run without pore volume iterations
  - In contrast to results from industry standard coupled simulations, the reservoir state is continuous and well-defined at all times.