A Novel Approach to Compaction Modeling in Sand / Sandstone Reservoirs

Øystein Pettersen
CIPR

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Motivation

- Rock compaction and fluid expansion are the two major sources of energy during primary depletion
- If compaction is irreversible, additional energy supply is provided during injection pressure build-up
- Understanding compaction mechanisms prior to production planning makes it possible to tune injectors optimally.
- Present situation: Compaction often initially defined as linear elastic, may later be modified by production history
  - Historical decline data cannot determine if compaction is irreversible, in which case injection performance will be dramatically improved.

Governing Equations (simplified)

(1) Darcy's law

$$u_l = -\overleftrightarrow{K}(\overleftrightarrow{\sigma})\lambda_l \nabla \Pi_l$$

(2) Conservation of mass

$$\nabla \cdot \left(\overleftrightarrow{K}(\overleftrightarrow{\sigma})\lambda_l^* \nabla \Pi_l \right) + \nabla \cdot \left(\overleftrightarrow{K}(\overleftrightarrow{\sigma})\lambda_o^* R_s \nabla \Pi_o \right)\delta_{gl} + Q_l = \frac{\partial}{\partial t}\left[\phi(\overleftrightarrow{\sigma})(b_l S_l + b_o S_o R_s \delta_{gl}) \right]$$

(3) Elastic strains

$$\varepsilon_{ij} = \frac{1}{2}\left(\frac{\partial \varsigma_i}{\partial x_i} + \frac{\partial \varsigma_j}{\partial x_j} \right)$$

(4) Stress – strain relationship

$$\sigma_{ij} = 2G\varepsilon_{ij} + \left(\lambda\varepsilon - \alpha p_f\right)\delta_{ij}$$

(5) Steady State Rock Momentum balance

$$\nabla \cdot \overleftrightarrow{\sigma} + \mathbf{F} = 0$$

Definitions / shorthand

Mobility

$$\lambda_l = \frac{k_{rl}}{\mu_l}, \quad l = o, w, c; \quad \lambda_l^* = \frac{\lambda_l}{B_l}$$

Pseudopotential

$$\nabla \Pi_l = \nabla p_l - \gamma_l \nabla z$$

Lamé constants (E: Youngs modulus, ν: Poisson's ratio)

$$G = \frac{E}{2(1+\nu)}; \qquad \lambda = \frac{E\nu}{(1-2\nu)(1+\nu)}$$

Mean normal stress

$$\sigma = \frac{1}{3}\left(\sigma_{11} + \sigma_{22} + \sigma_{33}\right)$$

Volumetric strain

$$\varepsilon = \varepsilon_{11} + \varepsilon_{22} + \varepsilon_{33}$$

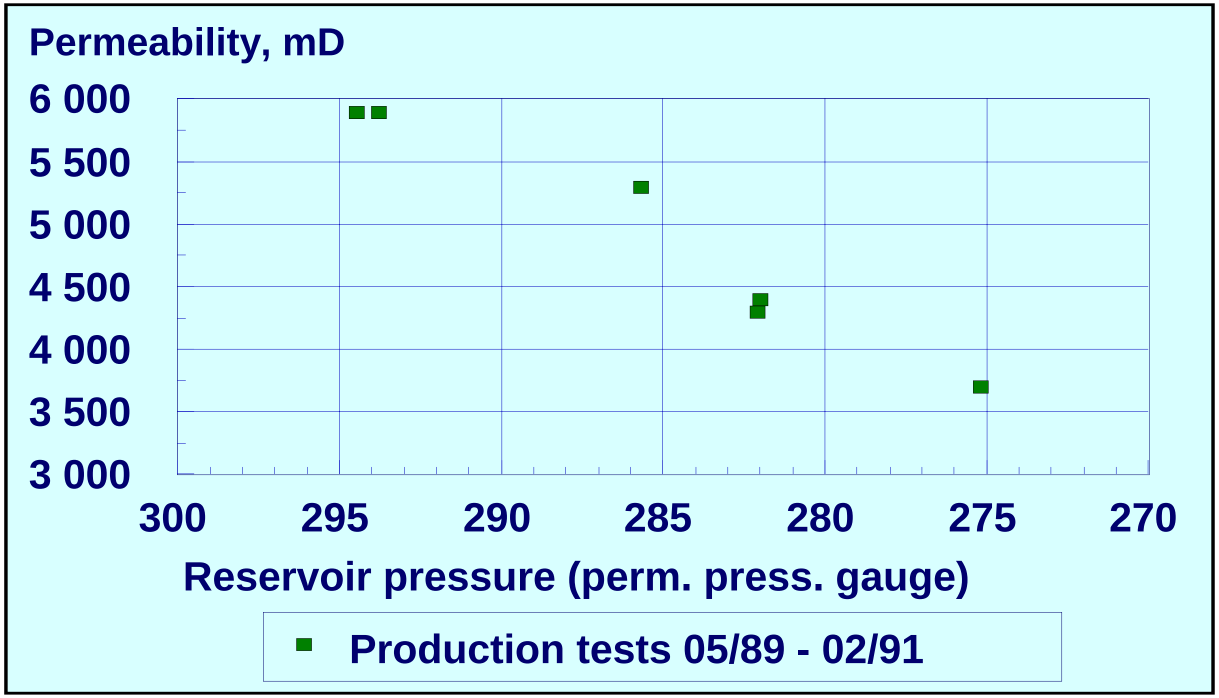
$$b_l = \frac{1}{B_l}$$

Coupling

- **Flow equations depend on the stress field since permeability and porosity are stress-dependent**
- **Stress-strain equations depend on fluid flow state through the fluid pressure term**
- **Fluid flow equations are implicitely coupled to stress through compaction, which may change bulk control volumes**

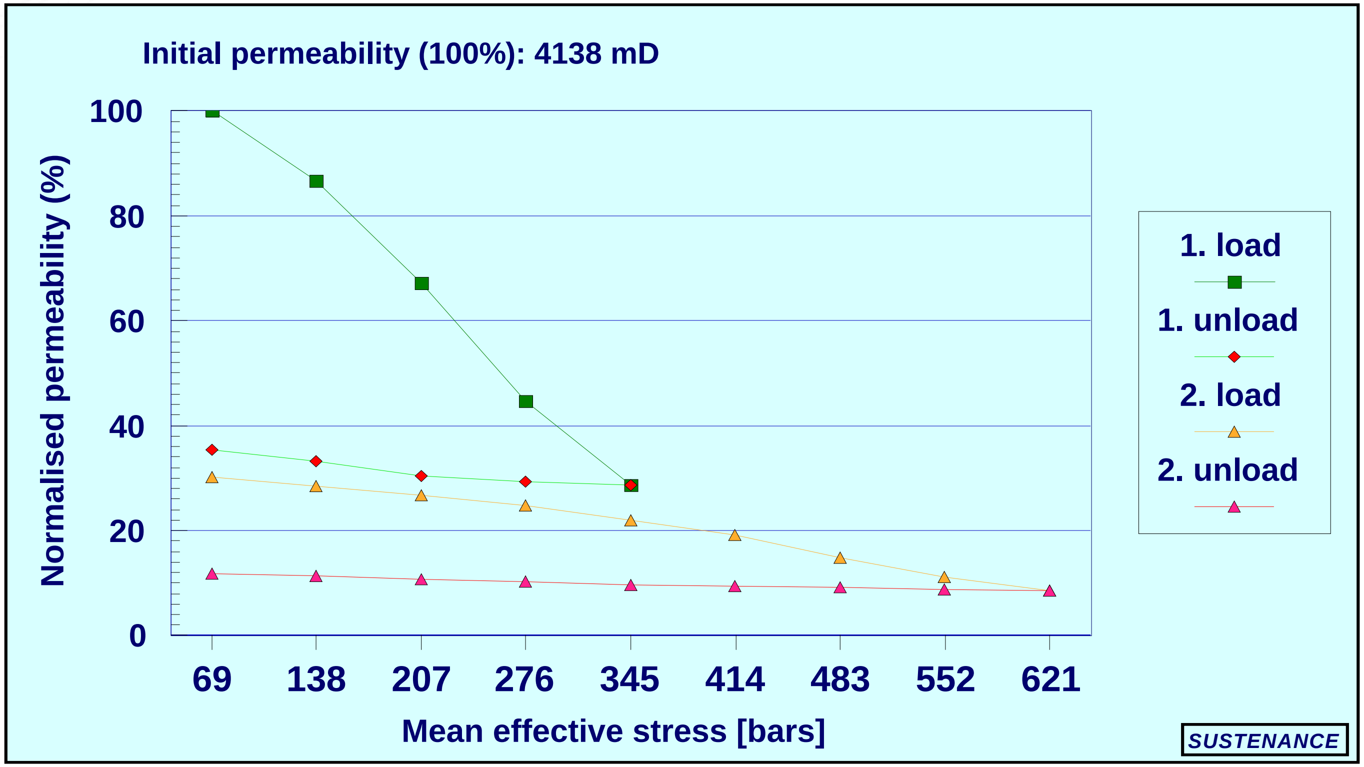
Examples of stress dependent permeability (1)

Permeabilities from transient test analysis (Gullfaks)

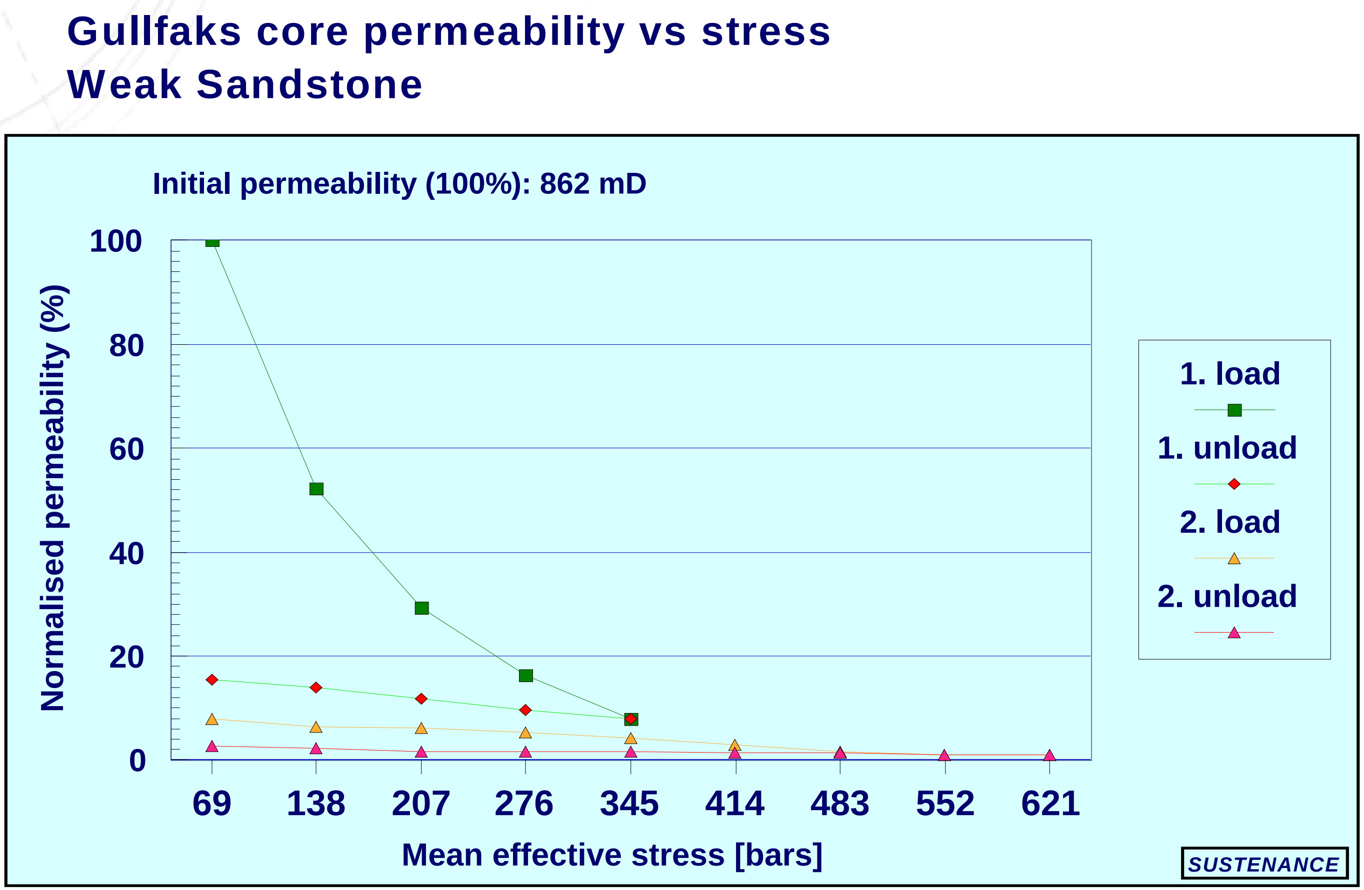


Examples of stress dependent permeability (2)

Gullfaks core permeability vs stress
Unconsolidated Sand

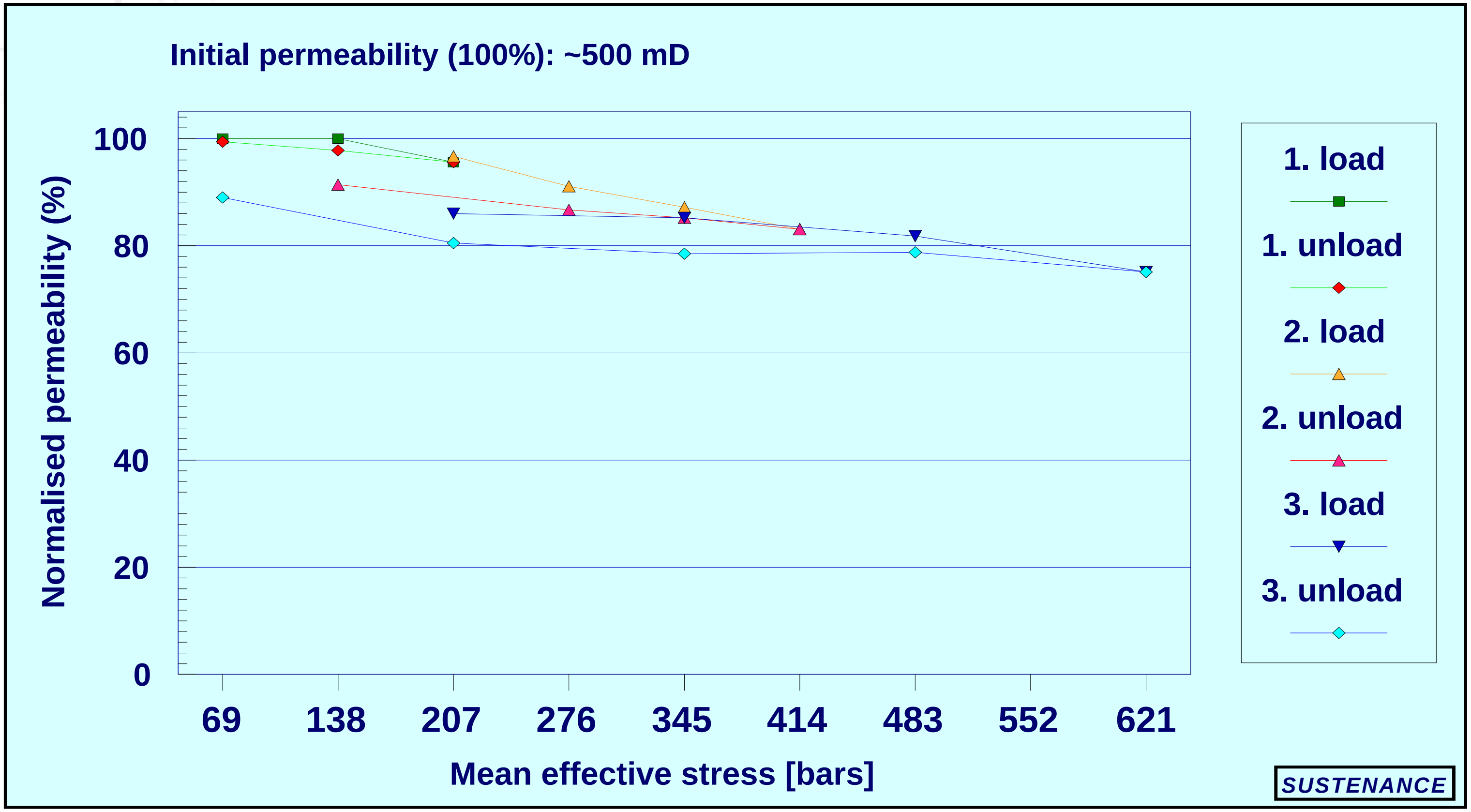


Examples of stress dependent permeability (3)



Examples of stress dependent permeability (4)

BP "F" field permeability vs stress
Weak Sandstone



Effective Stress

- In poro-elastic (or poro-plastic) materials, stress and fluid pressure are related by

$$\sigma' = \sigma - \alpha p_f$$

where σ' is effective stress, α is Biot's constant, and p_f is fluid pore pressure.

σ is "surrounding stress", acting on the rock skeleton (bulk volume), while p_f acts on the fluid-filled pore space.
For sandstone / sands, Biot's constant is close to or equal to unity.
Hence, σ' can be seen as a volumetric stress acting on the pore walls, and as such is responsible for pore wall stability.

It has been experimentally verified, and generally accepted that the soil / sand response is determined by the effective stress

Linear elastic compaction

Hooke's law:

$$\varepsilon_x = \sigma_x / E \quad \text{or} \quad E = \sigma_x / \varepsilon_x$$

(E is Young's modulus)

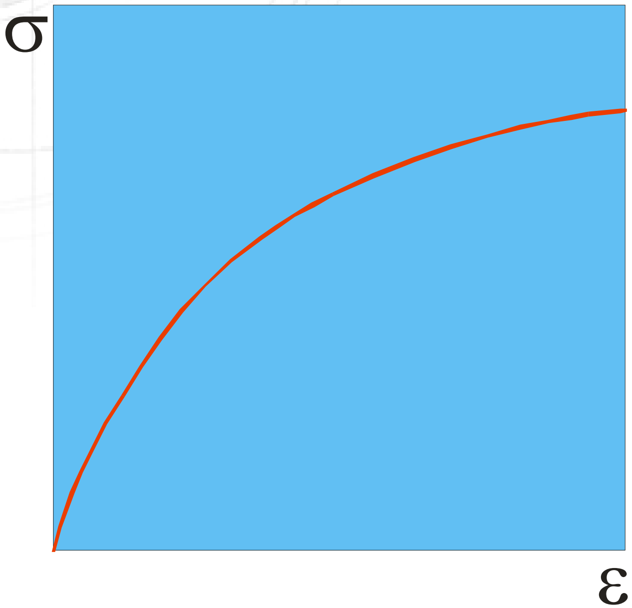
"No" poro-elastic samples obey Hooke's law,
experiments reveal that actually $E = E(\sigma)$.

Proposition: Materials deform according to Hooke's law,
but only on an incremental basis, i.e.

$$E = \frac{\delta\sigma}{\delta\varepsilon}$$

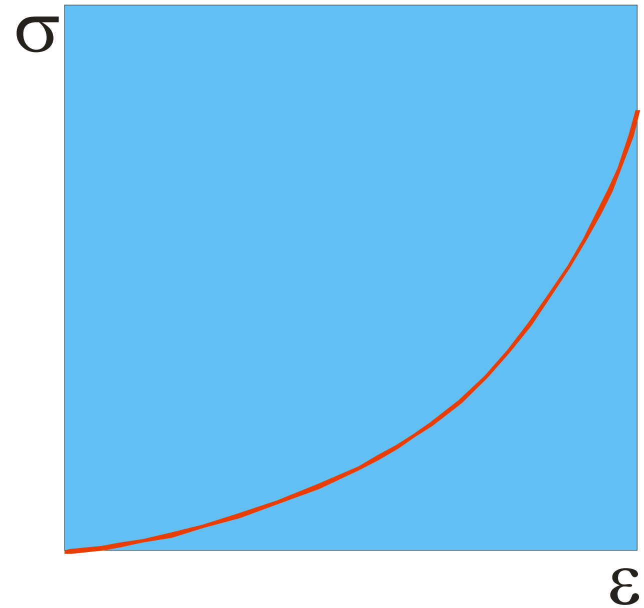
I.e., instead of interpreting nonlinear response as one material with varying E, each incremental change in σ changes the material itself, and the "new" material behaves incremental linearly with new elastic parameters.

Interpretation of stress – strain response



$$E, \frac{\delta\sigma}{\delta\varepsilon} \text{ decreasing}$$

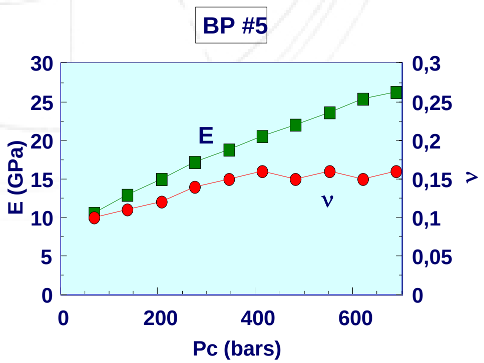
ε grows faster with increasing σ
Rock becomes weaker

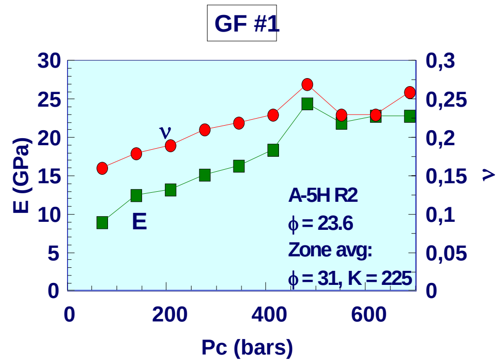


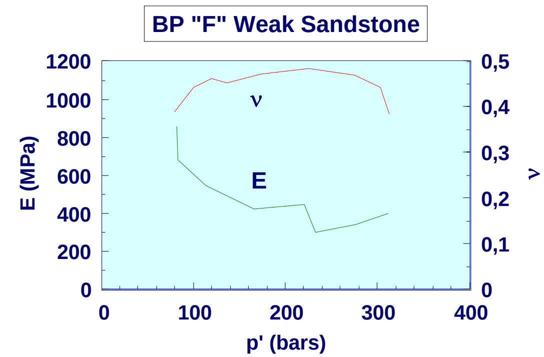
$$E, \frac{\delta\sigma}{\delta\varepsilon} \text{ increasing}$$

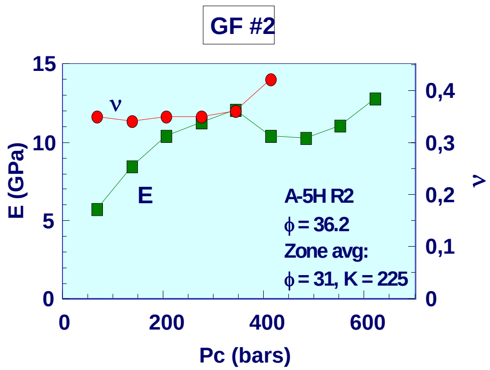
ε grows slower with increasing σ
Rock becomes stronger

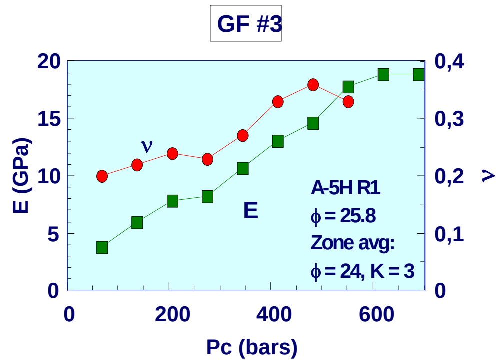
Examples of Young's modulus vs. stress

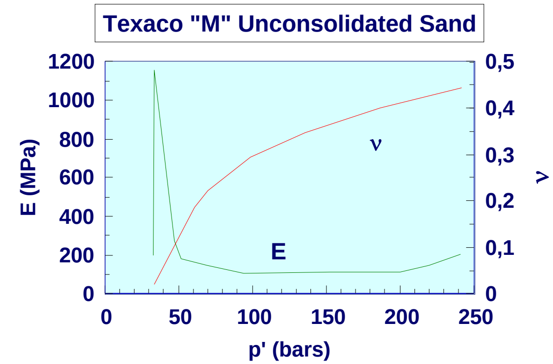












With a few exceptions, E increased with σ in all experiments i.e. in general the sand(stone) grows stronger during loading

Part I. Poro-elasticisity

Grain compaction

- E is (at least) an order of magnitude larger for the grains than for the porous sand. Hence very little of observed compaction can be attributed to grain compaction.
  - Typical examples (E, GPa):

Uncons. sand.	0.01 – 0.1
Sandstone	0.1 – 30
Shale	0.4 - 70
Granite	5 - 85

- Standard assumption in poro-elasticity / poro-plasticity: **Grain compaction is neglected in comparison with pore compaction**.
- Grain crushing can occasionally occur (e.g. shear stress movement can ”break off a corner”)

Grain packing

- Pore space is defined by the void between packed sand grains. As a change in pore volume is *not* the result of grain compaction, a reduction in pore volume can only be explained by tighter packing of grains.
- Observation 1: Grains are not maximal packed at reservoir initial conditions. (If they were, pore compaction would not be possible)
- Observation 2: Present grain packing is a result of the maximum stress level on the reservoir during its history (Stress Path)   (+ diagnesis, cataclasm,…)
- Grain reconfiguration will tend towards more stable packing.
  Corollary: Permanent deformation is probably the rule, reversible compaction is an exception.

Mechanisms of Compaction

Linear elasticity theory is applicable, at least for incremental strains.
In a geomechanical setting:

$$\delta V_p = -c_r V_p \delta\sigma'$$

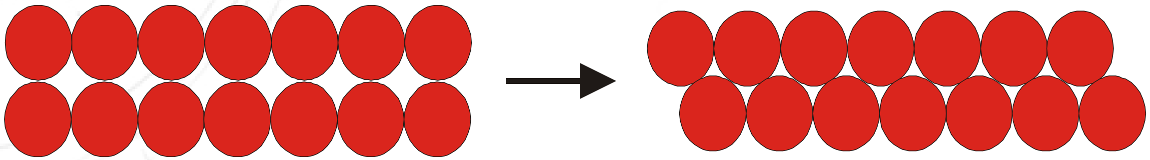
Reservoir simulators (e.g. Eclipse) uses an alternative formulation

$$\delta V_p = c_r V_p \delta p_f$$

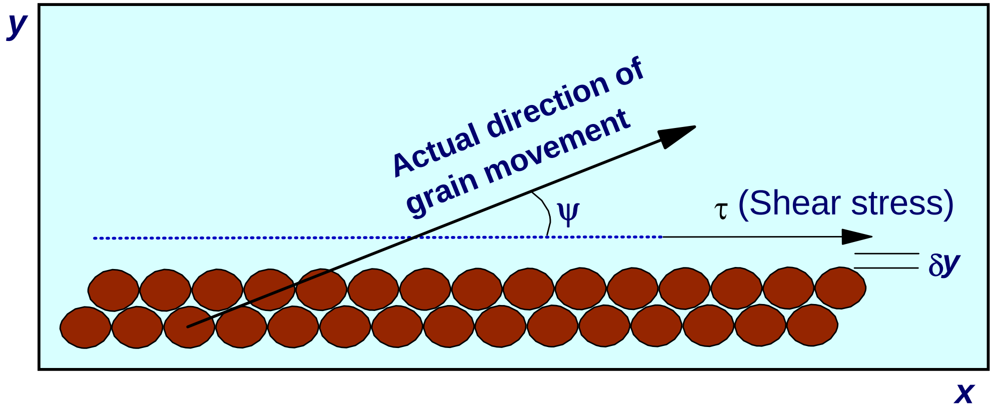
As formulated, the last expression has an implicit (but probably wrong) assumption that stress is constant during the compaction, and then the two are equivalent.

However, increased external stress would be due to e.g. subsidence, while reduced pore pressure doesn't necessarily affect the bulk volume. (Naïve first order assumption…)

Pore space collapse



Pore space reconfigures to more stable packing.
Further packing will be harder to achieve (E has increased).
It is improbable that unloading will regain previous packing.
However (in rare situations) shear stress can induce dilatancy, as exemplified below.



Pore space collapse 2

- Would intuitively be described as a pore wall *failure*
- Geomechanically a contradiction, as the material cannot fail as it grows stronger
- Failure on pore level, but poro-elastic on a larger level?
- How does the pore wall fail in general?
  - New stable (more stable) grain packing?
  - Sand production?
  - Chemical effects?
- Is Coulomb hypothesis (e.g. Mohr-Coulomb circles) applicable?

Pore space collapse 3

- Disregarding fluid transport of grain particles, control volume mass is constant.
- Bulk volume V_B consists of solid volume V_s and pore volume V_P
  - During compaction, $\Delta V_s = 0$, such that $\Delta V_B = \Delta V_P$
  - If other reservoir regions don't expand (not possible if irreversible), the reservoir as a whole must compact
  - Consequence: Confining rock must expand or move. Sideburden expansion is not probable, and from experience underburden changes very little.
  - Conclusion: Subsidence or overburden swelling is a necessary consequence of irreversible reservoir compaction

Pore space collapse 4

- Collapse model suitable for unconsolidated sands (only?)
- Much experimental proof of (reversible, classic) poro-elastic materials exist, also linear.
- Since stable grain-packing is almost a physical axiom, reversible processes must act according to other rules than described here.
- Stronger skeleton seems a plausible explanation, whereby classification of skeleton strength becomes important.

Recommendations for elastic media

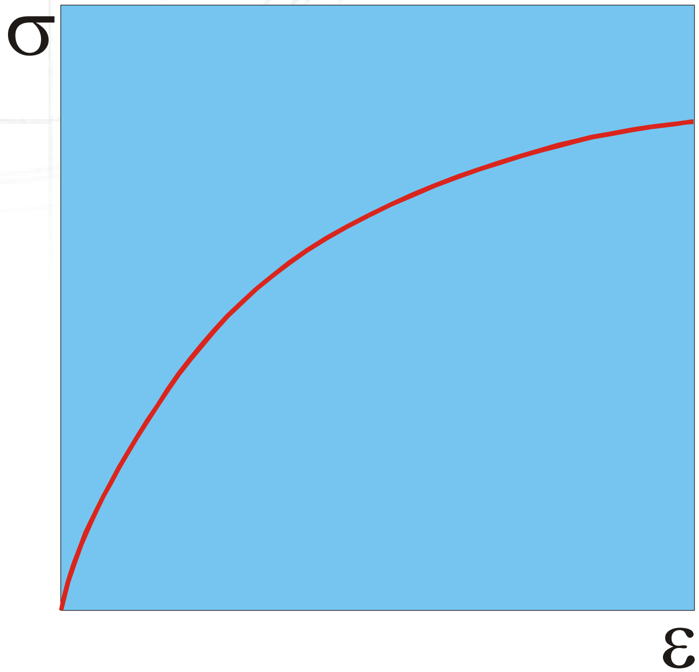
- Eclipse Compaction tables ($PORV(p_f)$) can be defined from stress – strain curves or by history matching. For practical purposes the interpretation is insignificant (non-linear curve or incremental linear), however
  - The standard nonlinear curve should not be treated as elastic (which would mean it was reversible). In most sand / sandstone fields each increment of strain defines a permanent deformation at that state, some times hysteresis may be more appropriate.
  - When constructing compaction tables from history data, the irreversible stress – strain relationship must be isolated. E.g. aquifer interaction can give apparent compaction, but will be a reversable effect.
  - Permeability will very often be compaction dependent

2Do

- Study relationship between Young's modulus (magnitude, dependence on σ) and other reservoir parameters (K, ϕ, burial depth,…)
- Coupled stress-flow simulations with systematic variation of relevant reservoir parameters to reveal mechanics of pore collapse.
  - Conditions for stable / unstable grain packing
  - Sand production
  - Other effects that change mass of solid in control volume
- Examine procedures for establishing (Eclipse) compaction curves and conditions for reversibility without performing (coupled) simulations or need for history data.

Part II: Poro-plasticity

Rock weakening

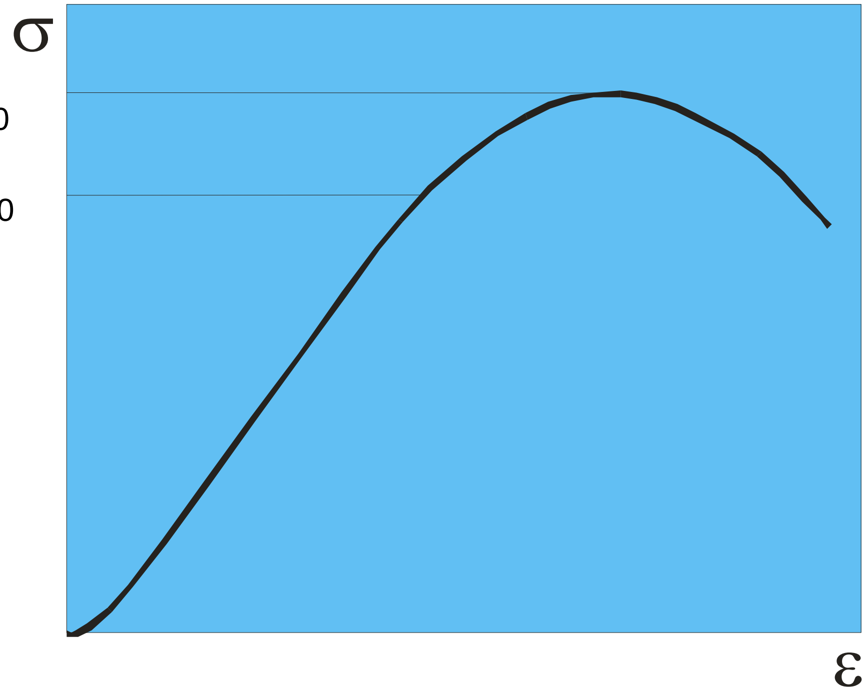


ε/σ cannot increase indefinitely. (There is a limit for how weak the rock can get.)
At some stressvalue the corresponding strain would be unphysically large, so "something" happens to the sample at a critical value of σ

Soil strength

Uniaxial compressive strength C_0

Yield point σ_0



Following curve from the origin: Elastic region until curve starts deflecting. First sign of weakness is defined as *yield point*. Advancing to *peak stress* we pass through the *ductile* region, where material has ability to endure permanent deformation without losing strength. Lastly the *brittle* region is entered, where in practice the material fails.

Note: When yield point is reached, the material properties change ("new" material)

Nomenclature

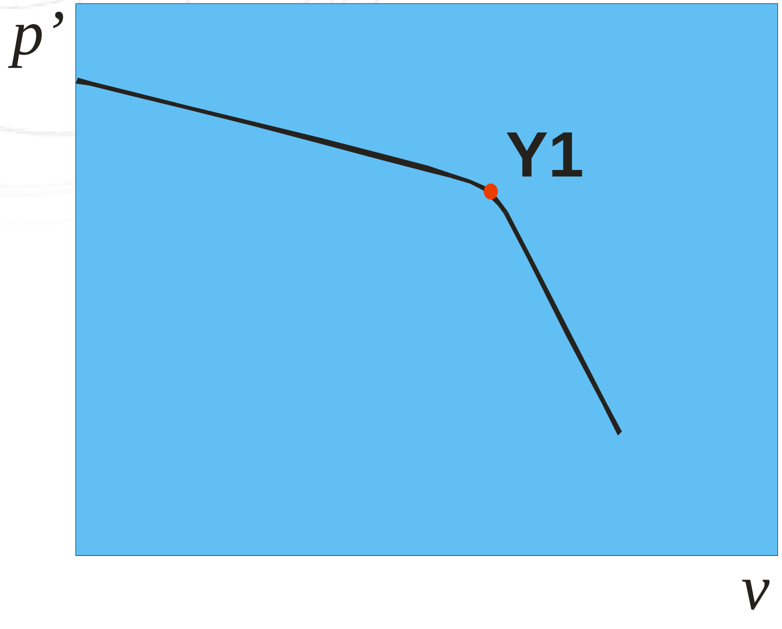
Volumetric stress: $p' = \frac{1}{3}\left(\sigma'_{xx} + \sigma'_{yy} + \sigma'_{zz}\right)$

Deviatoric stress: $q = \sigma_I - \sigma_{III}$

(σ_I and σ_{III} are smallest and largest stress components)

Specific volume: v

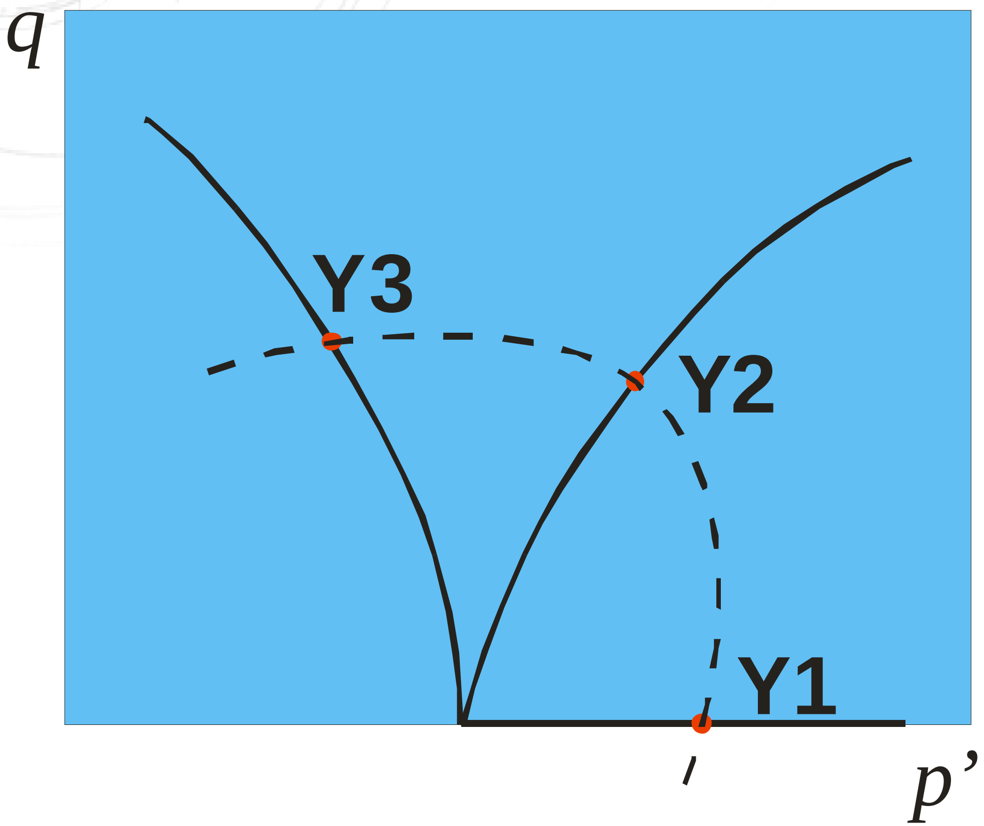
Experimental determination of yield point



Experiment can e.g. be performed on three *identical* samples, with differing conditions
1) Increasing p_f, σ constant
2) Drained compression
3) Undrained compression

Will provide three different yield points, all indicative of $(p' : q)$ combinations where material will yield.

Yield surface



The p' : q – paths of experiments have been plotted in p' : q – plane, and measured yield points Y1, Y2 and Y3.
These three points indicate a yield curve for the material (dashed)

In p' : q – space we would have a yield surface.

The yield surface is a boundary for elastically attainable states.

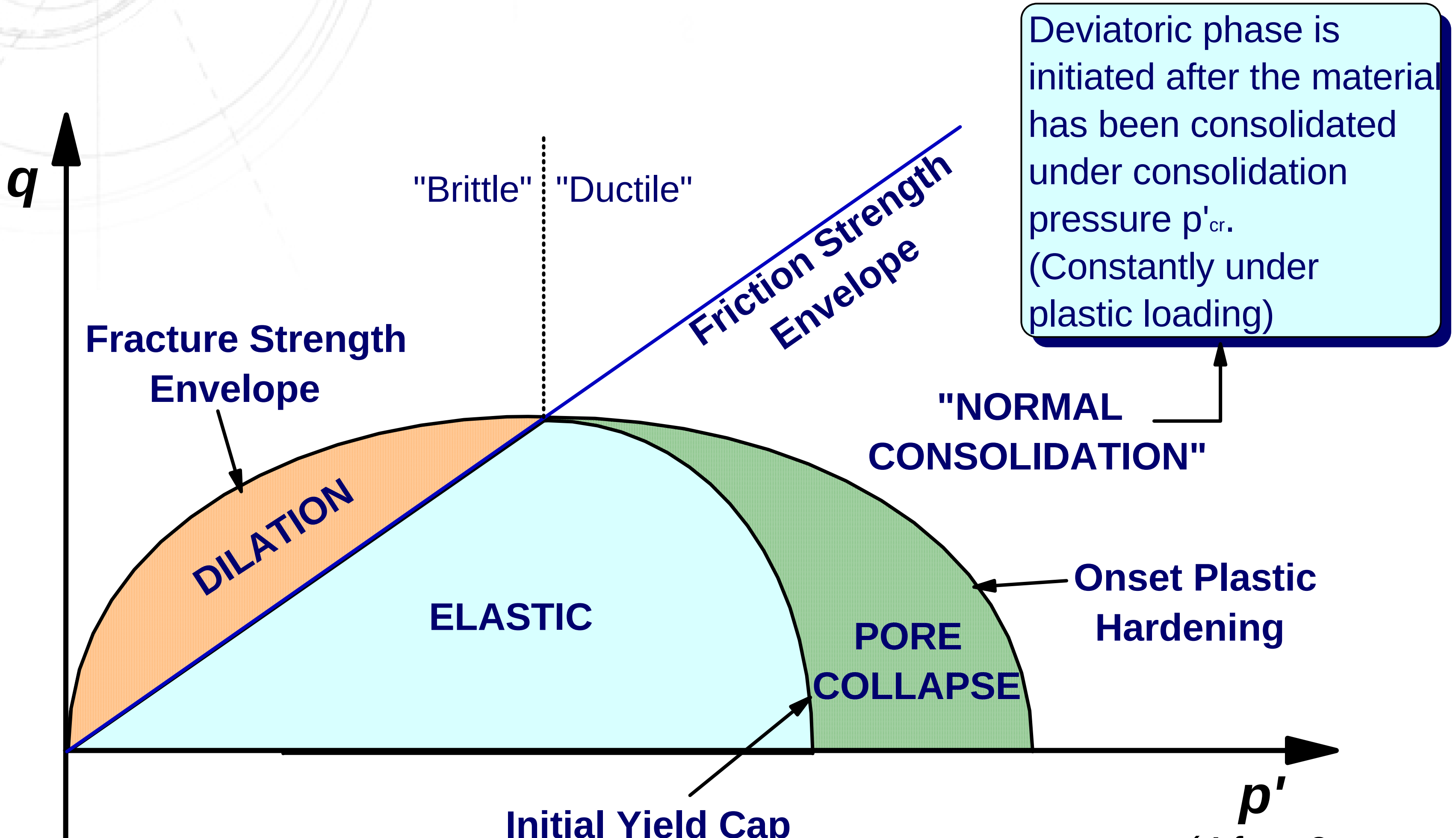
Development of yield surface

- When the stress state touches the yield surface, the material properties change, and this ”new” material has its own yield surface, necessarily outside the original.
- The ”new” material carries no information about its past
  - It is impossible to determine previous stress states for a material from its present state
- Often the different yield surfaces are regarded as one evolving surface associated with the original material.
- When the material yields, and enters the ductile region, further strain / deformation is determined by plastic flow rules. (These do not apply to the brittle region)

Critical State Theory

- For each yield curve (surface), the peak stress is recorded. The line through the peak stresses is called the critical state line (CSL).
- By the Cam Clay Model, deformation processes in poro-plastic materials will in some manner converge to the CSL and stay there
- Yield surfaces and CSL can be used to define the different material states in p’:q plane (space)

Deformation Processes in Cam Clay Model



(After Scott et. al.)

2Do

- Study the different parameters causing poro-plastic failure by coupled stress-flow simulations.
- Identify critical parameters which signify probable failure (extend existing theory)
- Establish rules of thumb for a priori recognition of types of failure from pure flow parameters (?)

Example, attempt at correlation

Cross Plot All Sources

Cohesion (S_0), Friction Angle (φ) vs. Porosity (ϕ),