Sandstone Compaction Modelling and Reservoir Simulation ++

by

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(extended version)

Research areas

- Compaction modelling
- Improved coupled simulation
  (rock mechanics ↔ flow simulation)
- Stress modelling in faults
  - Fault generation
  - Stress field in / close to faults
  - Dynamic sealing properties
  - Mesh issues: Refinement / Adaptive remeshing

Governing Equations (simplified)

(1) Darcy's law

$$u_l = -\overleftrightarrow{K}(\overleftrightarrow{\sigma})\lambda_l \nabla \Pi_l$$

(2) Conservation of mass

$$\nabla \cdot \left(\overleftrightarrow{K}(\overleftrightarrow{\sigma})\lambda_l^* \nabla \Pi_l \right) + \nabla \cdot \left(\overleftrightarrow{K}(\overleftrightarrow{\sigma})\lambda_o^* R_s \nabla \Pi_o \right)\delta_{gl} + Q_l = \frac{\partial}{\partial t}\left[\phi(\overleftrightarrow{\sigma})(b_l S_l + b_o S_o R_s \delta_{gl}) \right]$$

(3) Elastic strains

$$\varepsilon_{ij} = \frac{1}{2}\left(\frac{\partial \varsigma_i}{\partial x_i} + \frac{\partial \varsigma_j}{\partial x_j} \right)$$

(4) Stress – strain relationship

$$\sigma_{ij} = 2G\varepsilon_{ij} + \left(\lambda\varepsilon - \alpha p_f\right)\delta_{ij}$$

(5) Steady State Rock Momentum balance

$$\nabla \cdot \overleftrightarrow{\sigma} + \mathbf{F} = 0$$

Definitions / shorthand

Mobility $$\lambda_l = \frac{k_{rl}}{\mu_l}, \quad l = o, w, c; \quad \lambda_l^* = \frac{\lambda_l}{B_l}$$

Pseudopotential $$\nabla \Pi_l = \nabla p_l - \gamma_l \nabla z$$

Lamé constants (E: Youngs modulus, ν: Poisson's ratio)

$$G = \frac{E}{2(1+\nu)}; \qquad \lambda = \frac{E\nu}{(1-2\nu)(1+\nu)}$$

Mean normal stress $$\sigma = \frac{1}{3}\left(\sigma_{11} + \sigma_{22} + \sigma_{33}\right)$$

Volumetric strain $$\varepsilon = \varepsilon_{11} + \varepsilon_{22} + \varepsilon_{33}$$

$$b_l = \frac{1}{B_l}$$

Coupling

- Flow equations depend on the stress field since permeability and porosity are stress-dependent
- Stress-strain equations depend on fluid flow state through the fluid pressure term
- Fluid flow equations are implicitely coupled to stress through compaction, which may change bulk control volumes

Compaction Modelling

- **Reservoir Simulator**: Compaction is a function of fluid pressure, $C_r = C_r(p_f)$
- **Reality**: Compaction is a function of effective stress
  → the difference between (confining) total stress and fluid pressure
- Measure for compaction in a simulator grid cell, **Pore Volume Multiplier**,

$$PVmult = \frac{\text{current cell pore volume}}{\text{initial cell pore volume}}$$

Computing *PVmult*

- From reservoir simulator:
  $PVmult(p_f)$ from $C_r(p_f)$ (table look-up)

- From rock mechanics simulator
  $PVmult(\text{strain}) = \exp(-\Delta \text{ vol. strain})$

Fluid Pressure and Stress

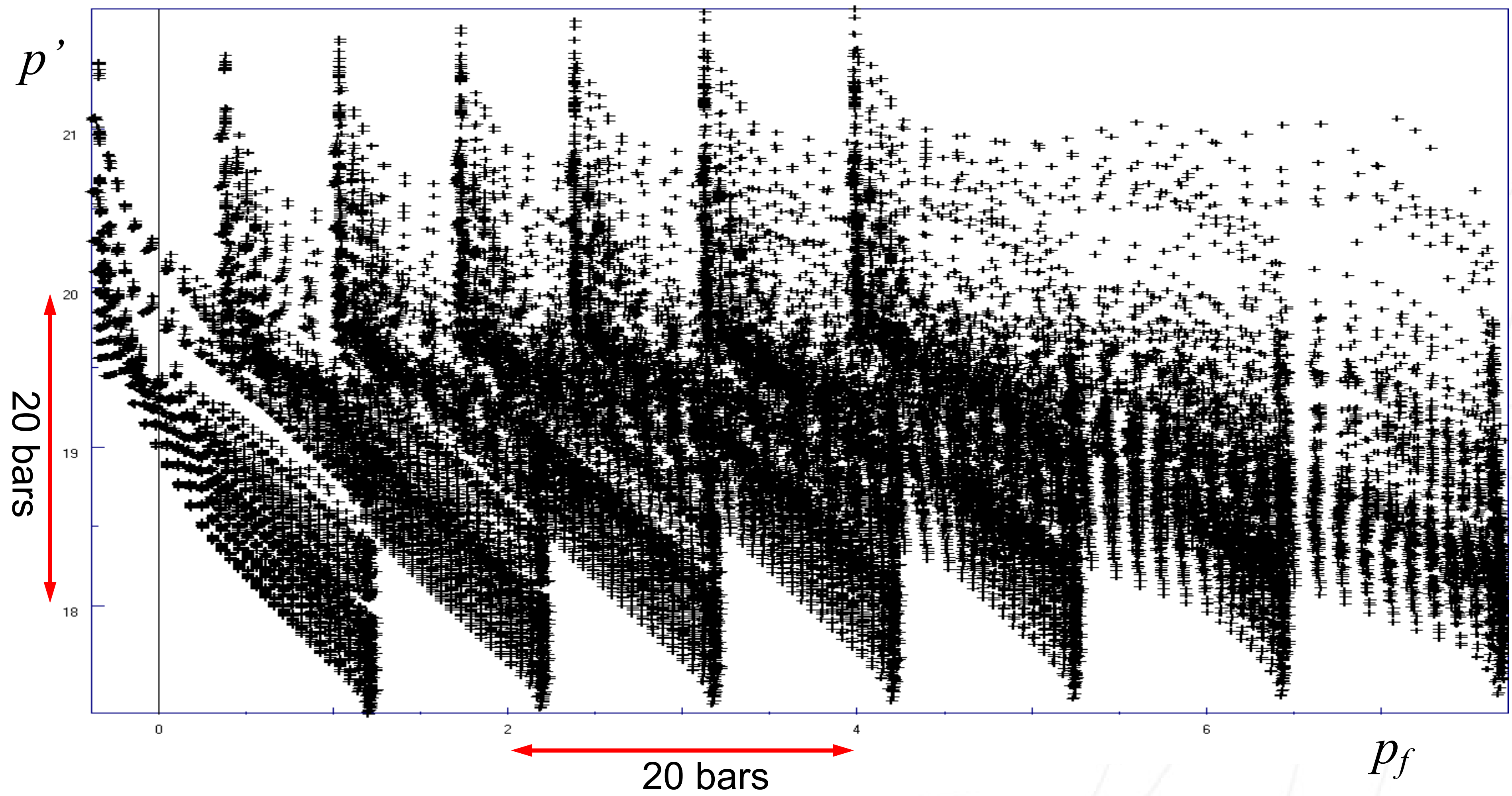
For practical purposes,

$$\sigma' = \sigma - p_f$$

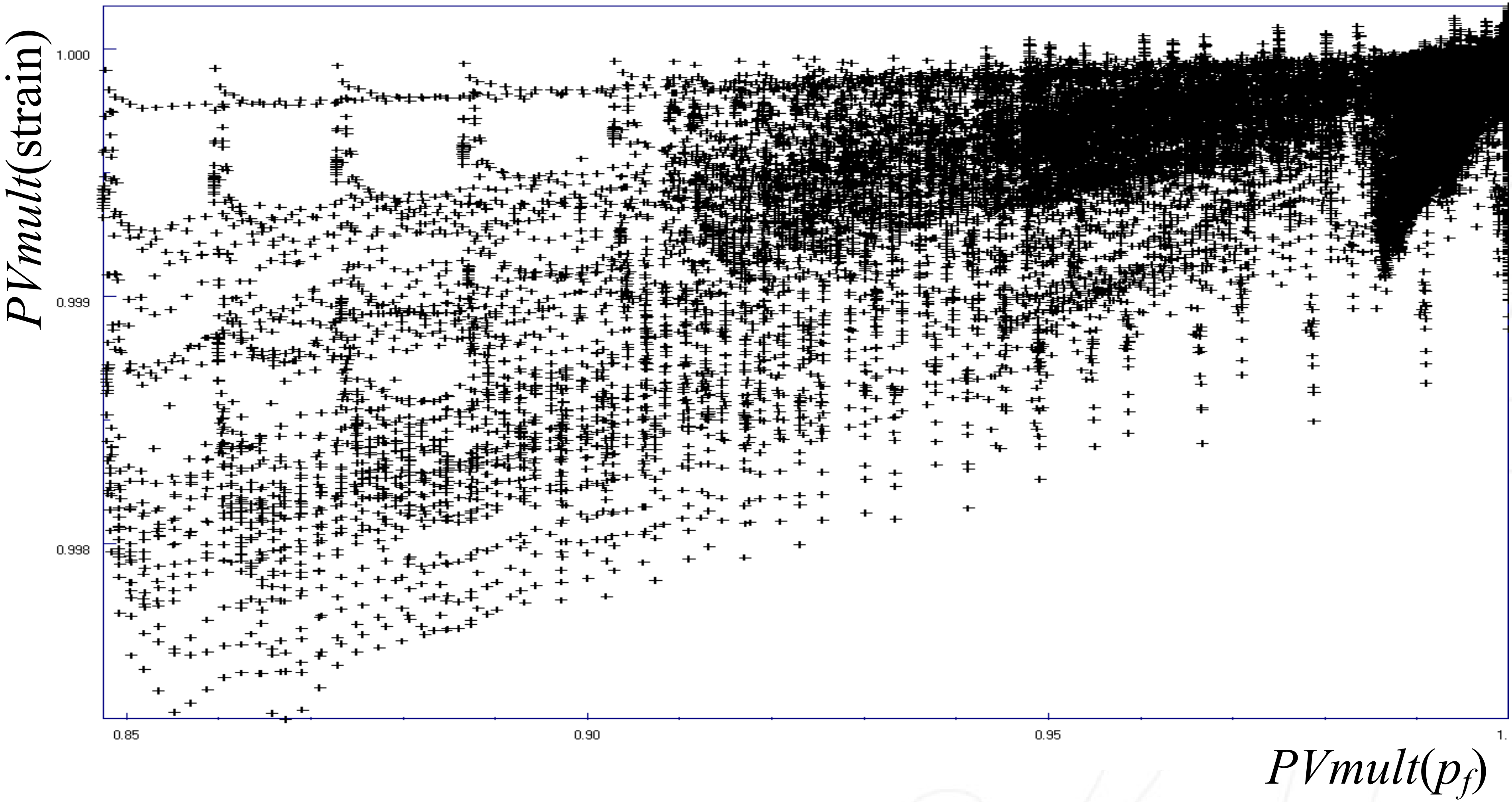
where
σ' and σ are effective and total stress
p_f is fluid pressure

I.e.: Assuming compaction is a function of fluid pressure
is equivalent to assuming total stress is constant,
or
mean effective stress vs. fluid pressure is a straight line

Correlation: Mean Eff. Stress vs. Fluid Pressure

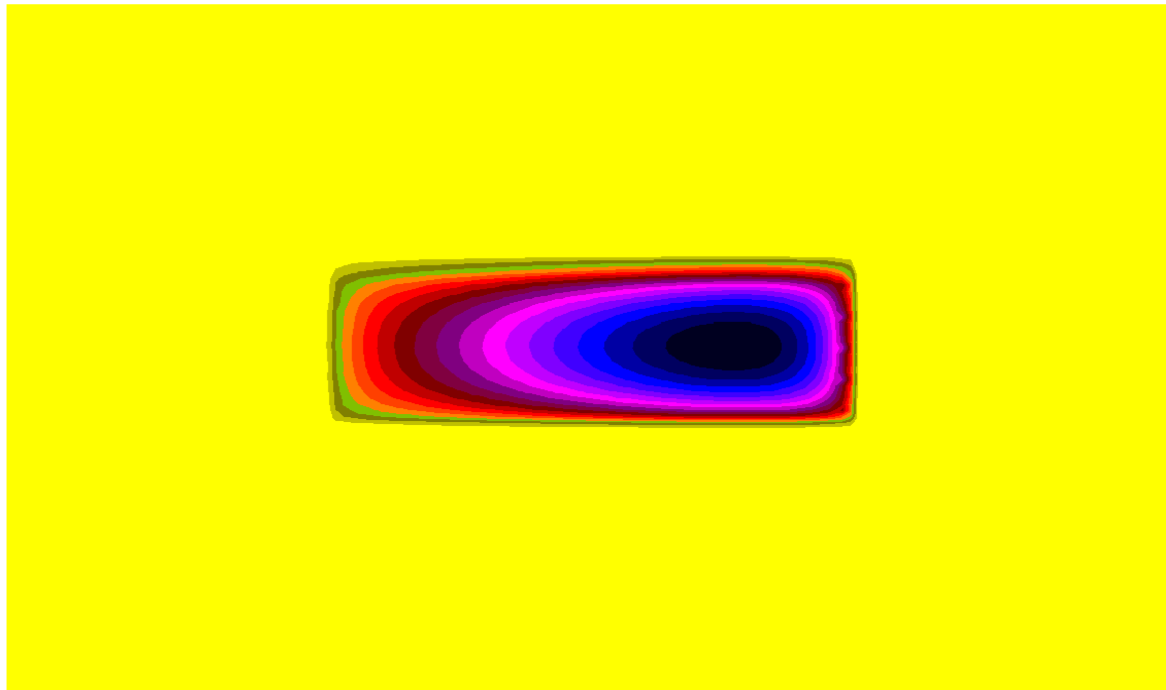


Correlation *PVmult*: "Correct" vs. from p_f



The main reason for the discrepancy is that the reservoir simulator knows nothing about actual soil displacement in the reservoir. (Boundary effects – “arching”)

$PVmult(p_f)$
(Reservoir only)



$PVmult(\mathrm{strain})$ (Reservoir and sideburdens)

Coupling Modes

- Fully coupled
  - Full system of fluid flow and rock mechanics equations solved simultaneously at each time step
  - Most accurate solution
  - Takes long to run
  - No fully coupled simulator includes all options that exist in commercial flow simulators or rock mechanics simulators

There is a growing awareness that

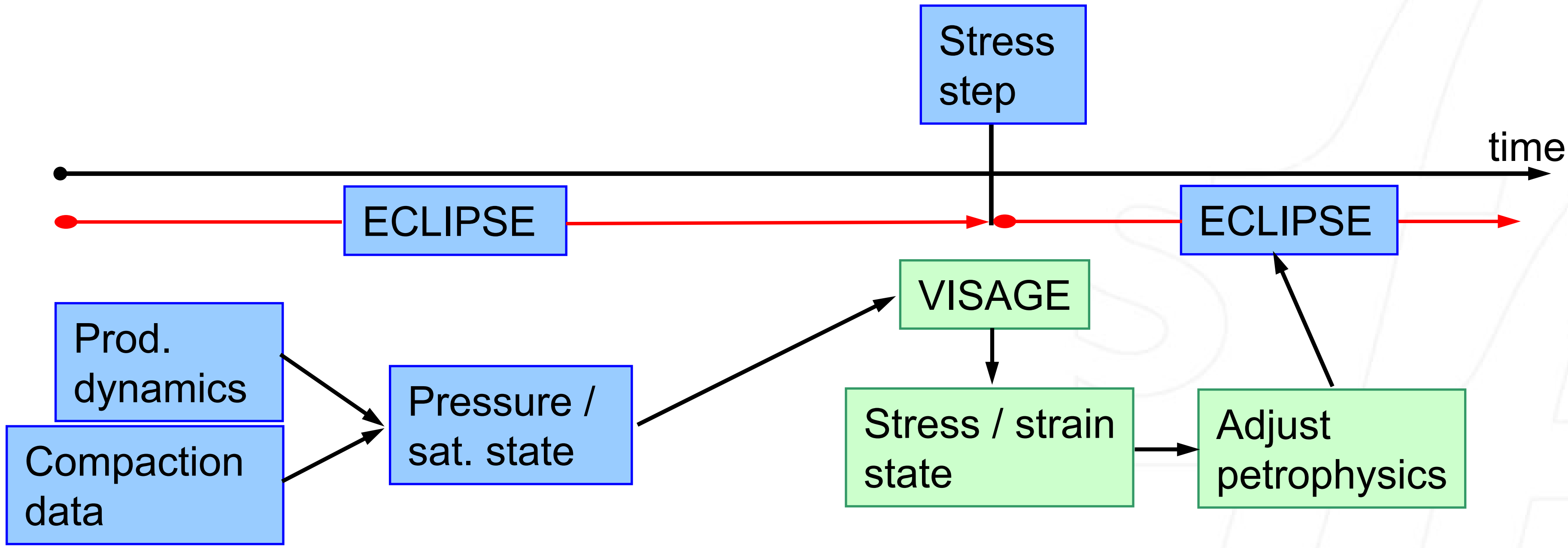
- Dynamic reservoir stress state often has significant influence on petrophysics and fluid production
- These processes can only be understood by performing coupled simulations
  (Rock mechanics simulator – Reservoir simulator)

In this study:

Finite Difference Reservoir simulator:
ECLIPSE from Schlumberger

Finite Element Rock Mech. simulator:
VISAGE from V.I.P.S Ltd.

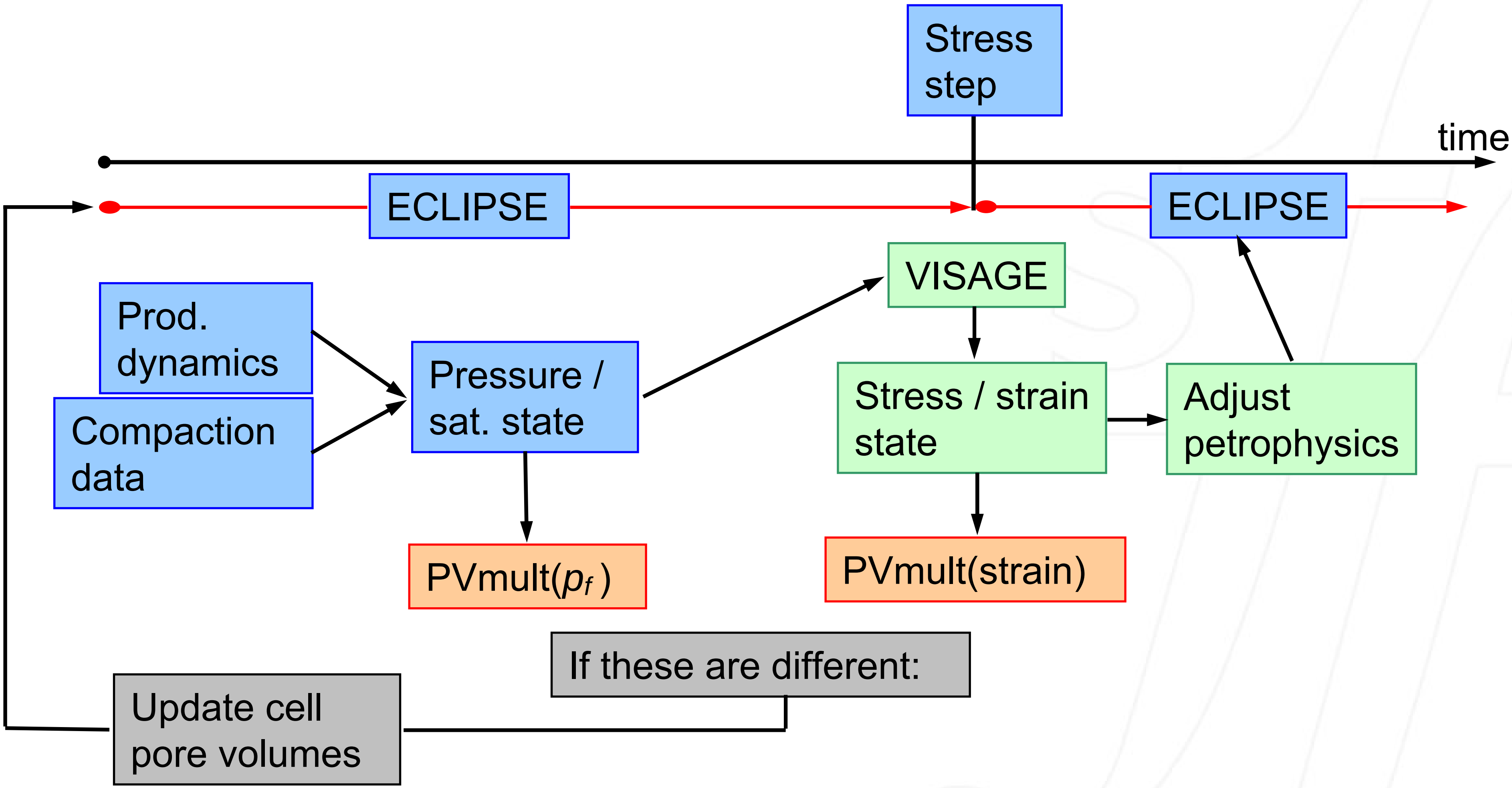
Coupling Modes: Explicit Coupling



Explicit Coupling

- Relatively fast
- Provides reasonably good reservoir stress state *distribution* (but not *level*)
- Questionable accuracy w.r.t. compaction modelling

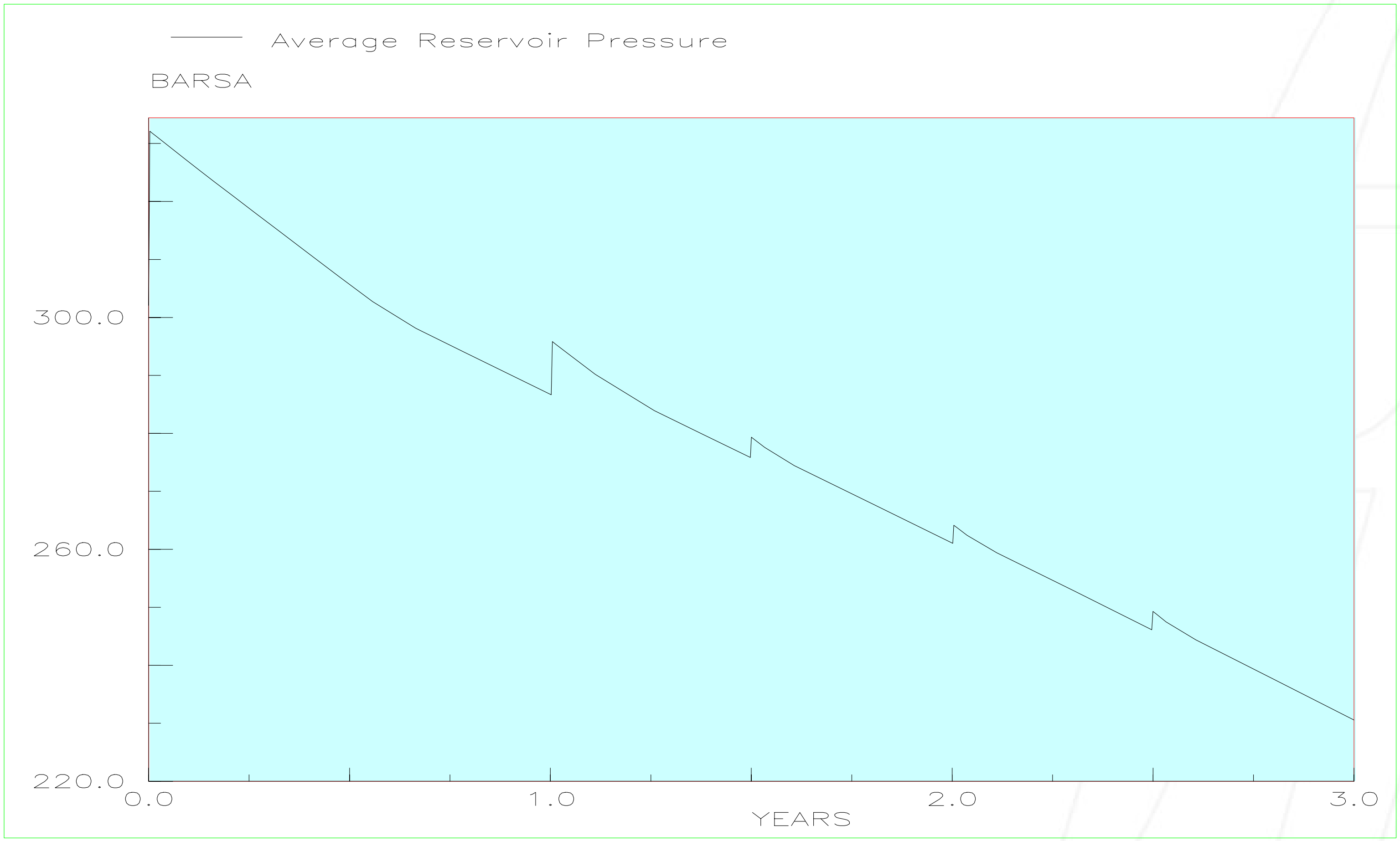
Coupling Modes: Iterative Coupling



Iterative Coupling

- Good reservoir stress state *distribution* and *level,*
  - Accurate compaction
- Can take long to run
- Updates performed only on stress steps
  - Pressure discontinuities

Pressure vs. Time in Iterative Coupled Run



Discontinuities at stress steps

Improved Coupling Scheme

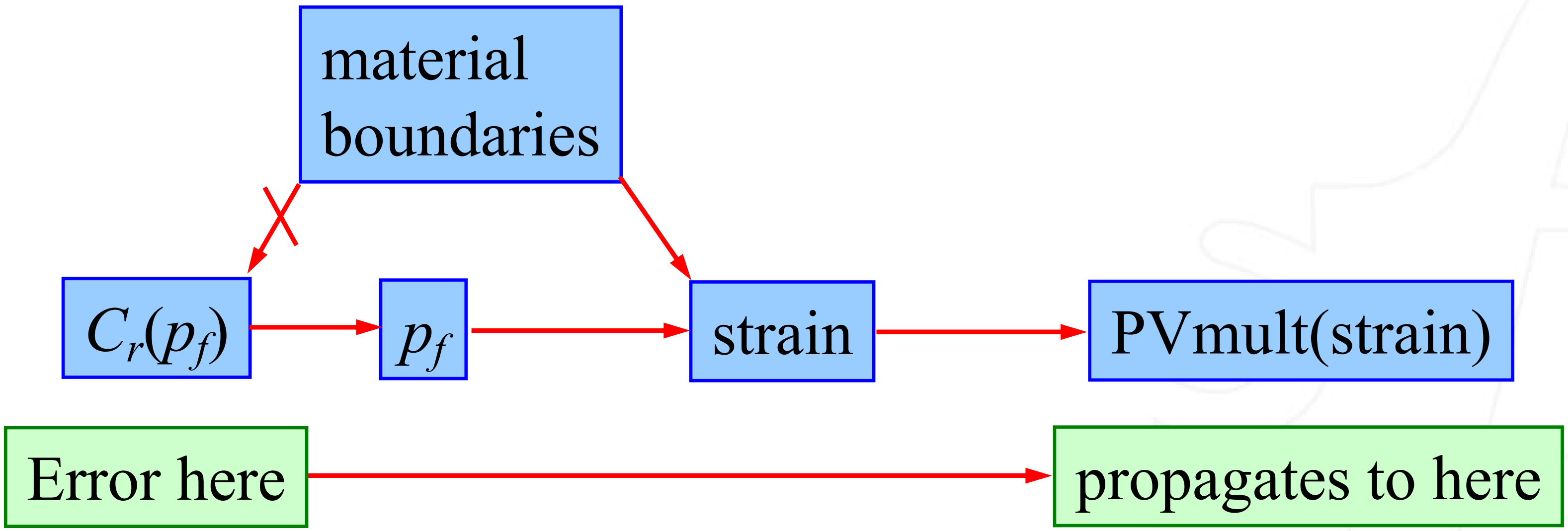
Calculation chain:

$\boxed{C_r(p_f)} \rightarrow \boxed{p_f} \rightarrow \boxed{\text{strain}} \rightarrow \boxed{\text{PVmult(strain)}}$

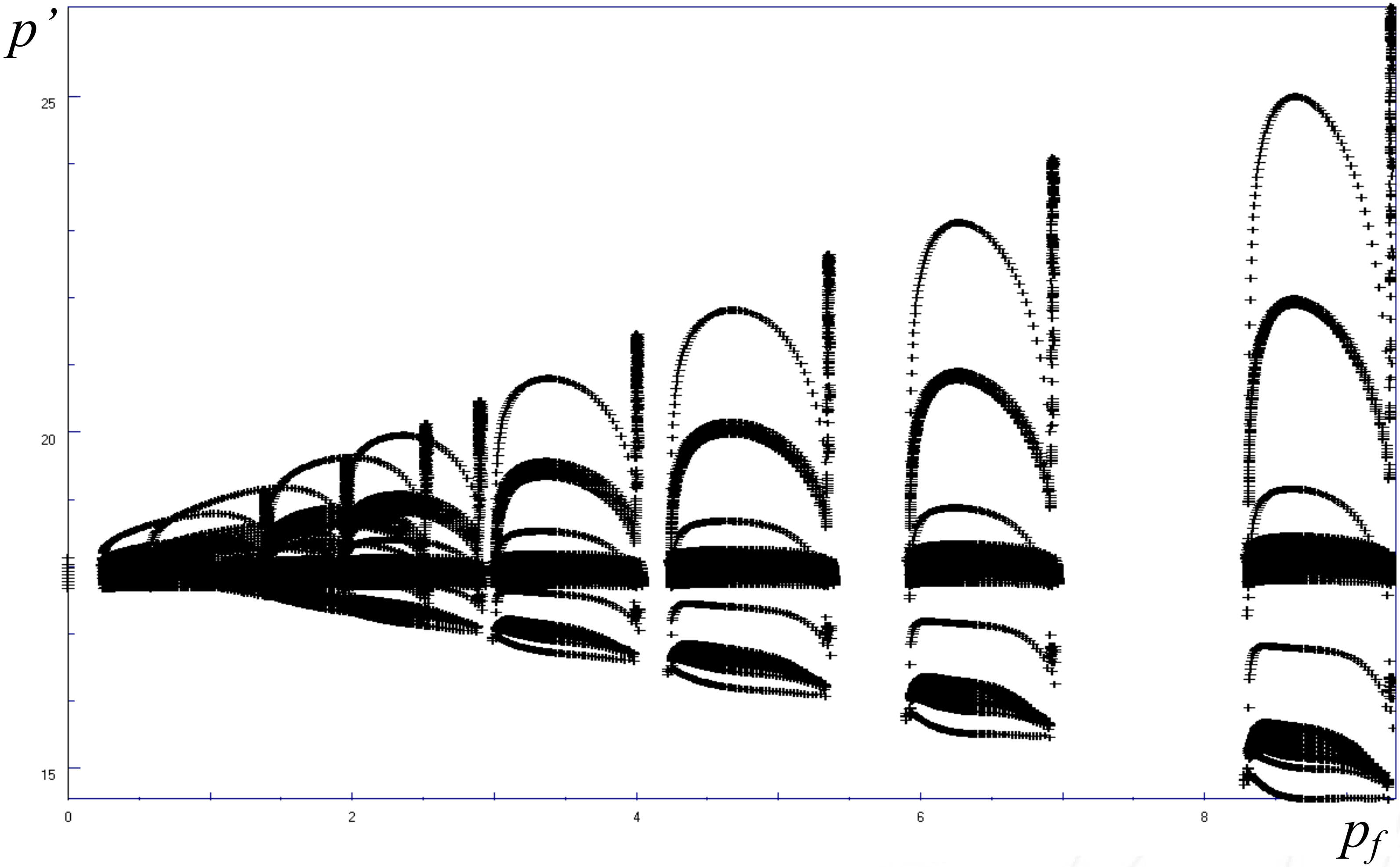
Reservoir simulator

Rock mech. simulator

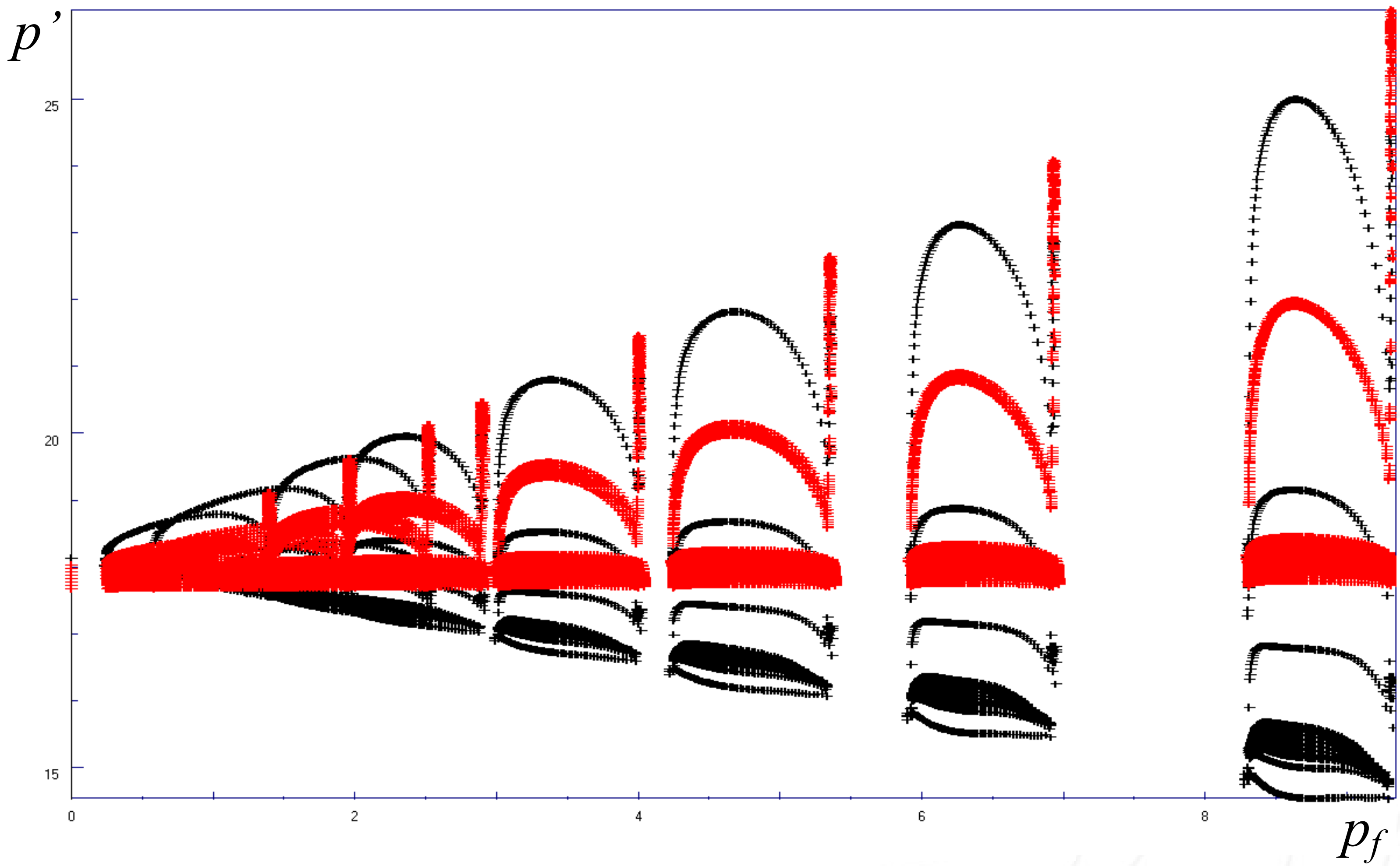
Improved Coupling Scheme



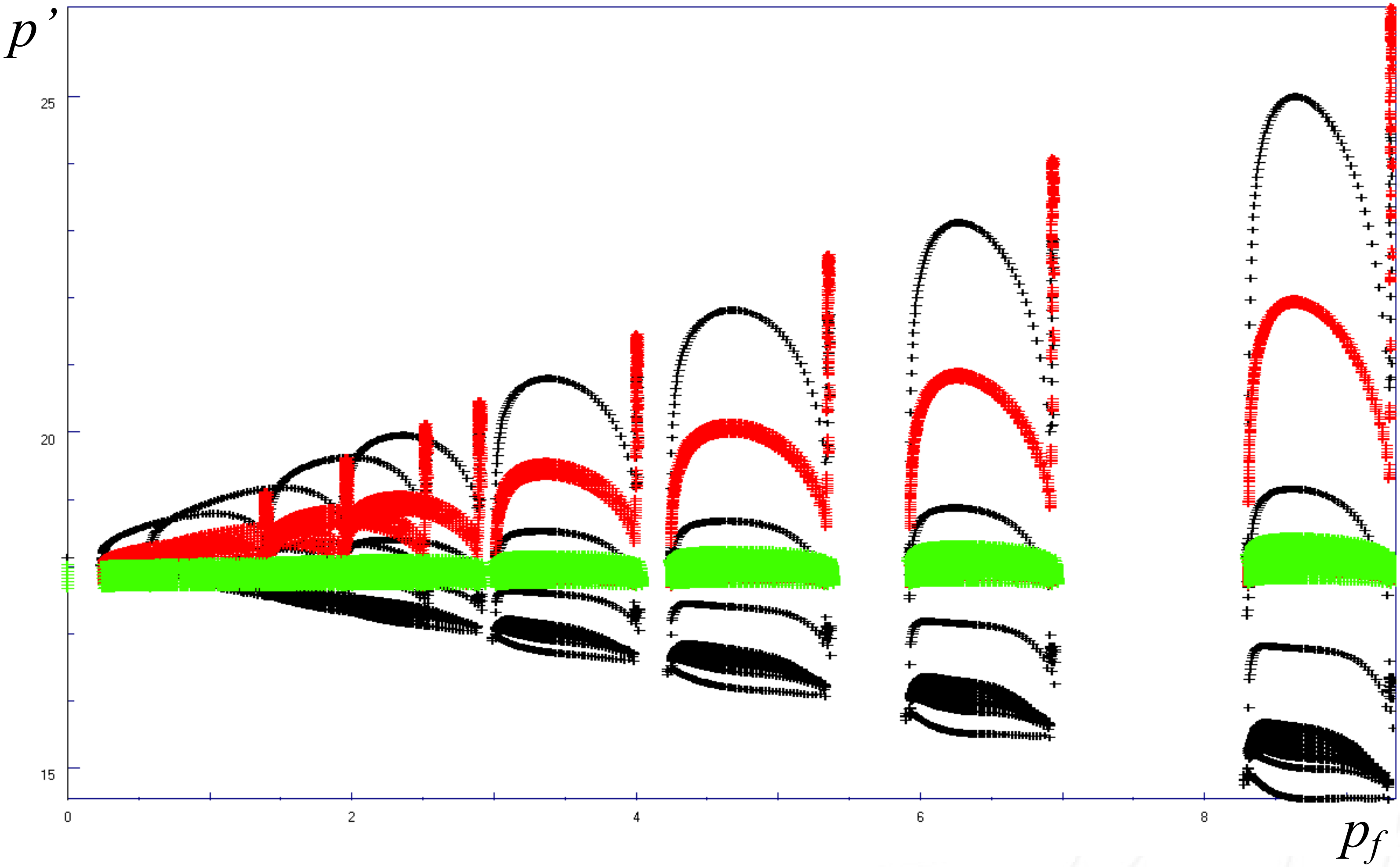
Correlation p' vs. pf, all cells



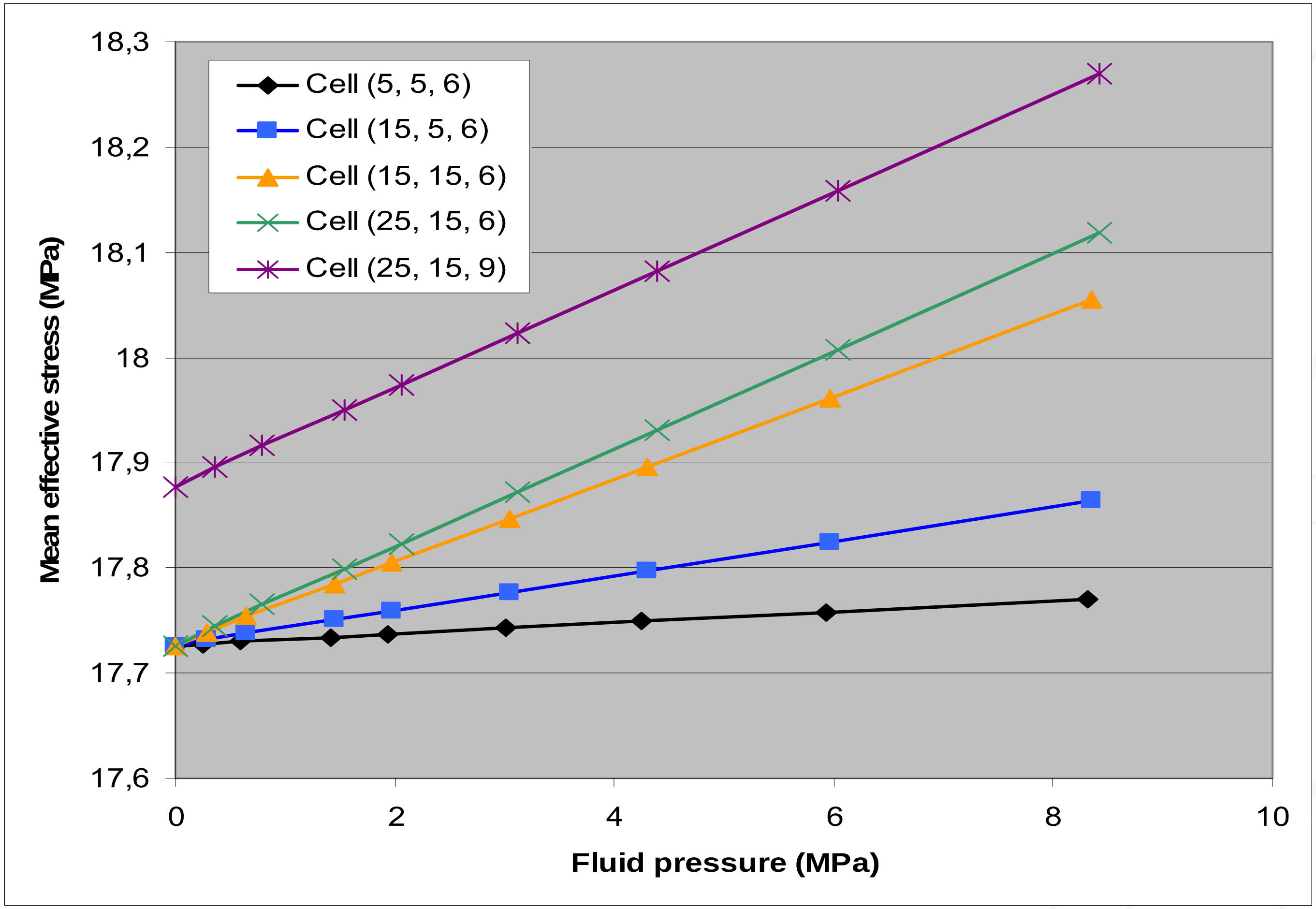
Correlation p' vs. pf, omit bottom layer



Correlation p’ vs. pf, omit boundary cells



Correlation p’ vs. pf at some cells



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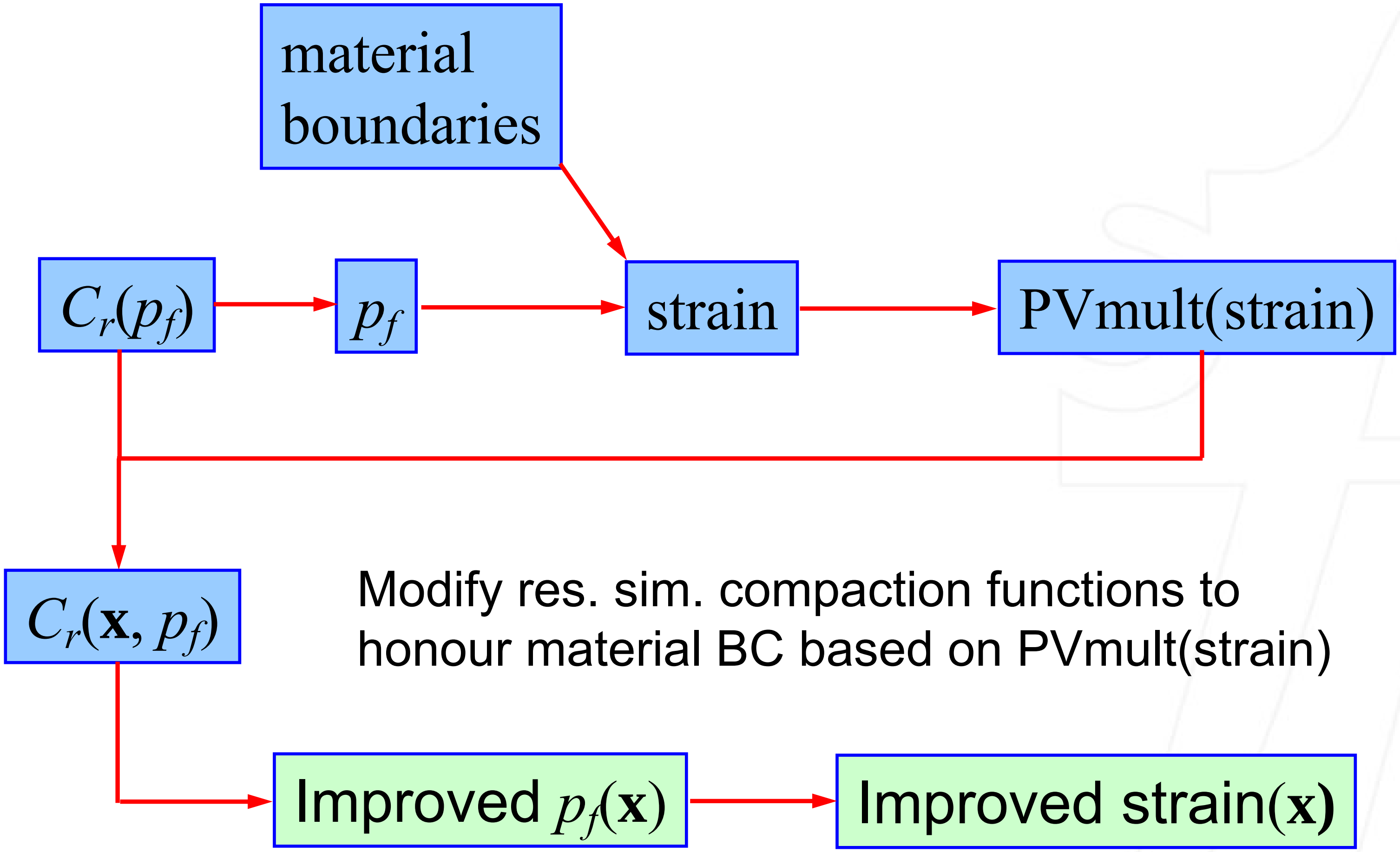

Modify res. sim. compaction functions to honour material BC based on PVmult(strain)

Improved Coupling Scheme

- extend trends in $C_r(\mathbf{x}, p_f)$ in space and time to get a better predictor for stress simulations
- Modifications done on res. sim. data: Cont. pressure

Goal

- ☺ Faster than iterative coupling
- ☺ More accurate than explicit coupling

Consistent Compaction Model

- To proceed we need a compaction model which is "equivalent" in VISAGE and ECLIPSE
- Definition:
  A compaction model is consistent if the flow simulator compaction function is derived from the rock mechanics poro-elasto-plastic model.

(Idealized) Grain Pack Model for Sand / Sandstone

Nomenclature

Compressibility $K = \dfrac{E}{3(1-2\nu)}$

Bulk Volume: V_B
Pore Volume: V_P
Solid Volume: V_S
Porosity: ϕ

Specific volume $v = \dfrac{V_B}{V_S} = \dfrac{1}{1-\dfrac{V_P}{V_B}} = \dfrac{1}{1-\phi}$

Mean eff. stress: p'
Deviatoric stress: q

Volumetric strain: ε_p

Basis – Pure Geomechanical Compaction

- Typical measured bulk compressibilities for sands / sandstones are much smaller than grain compressibility
  - ✓ Grain (quartz): K ~ 38 GPa
  - ✓ Sands: K = 100 MPa – 1 GPa
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- Principle of Stable Settlement:
  When grain packing changes, it will always seek a more stable packing pattern.

Consequences

- Each stress state corresponds to a stable packing configuration
  - the tightest possible packing at that state

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- As packing becomes tighter, further packing will be increasingly more difficult to achieve
  - each "packing level" is more stable than previous levels
  - Compressibility increases with load

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  - the tightest possible packing at that state
- As packing becomes tighter, further packing will be increasingly more difficult to achieve
  - each “packing level” is more stable than previous levels
  - Compressibility increases with load
- Relieving stress will *not* return the soil to a previous, *less stable* packing level

Implications

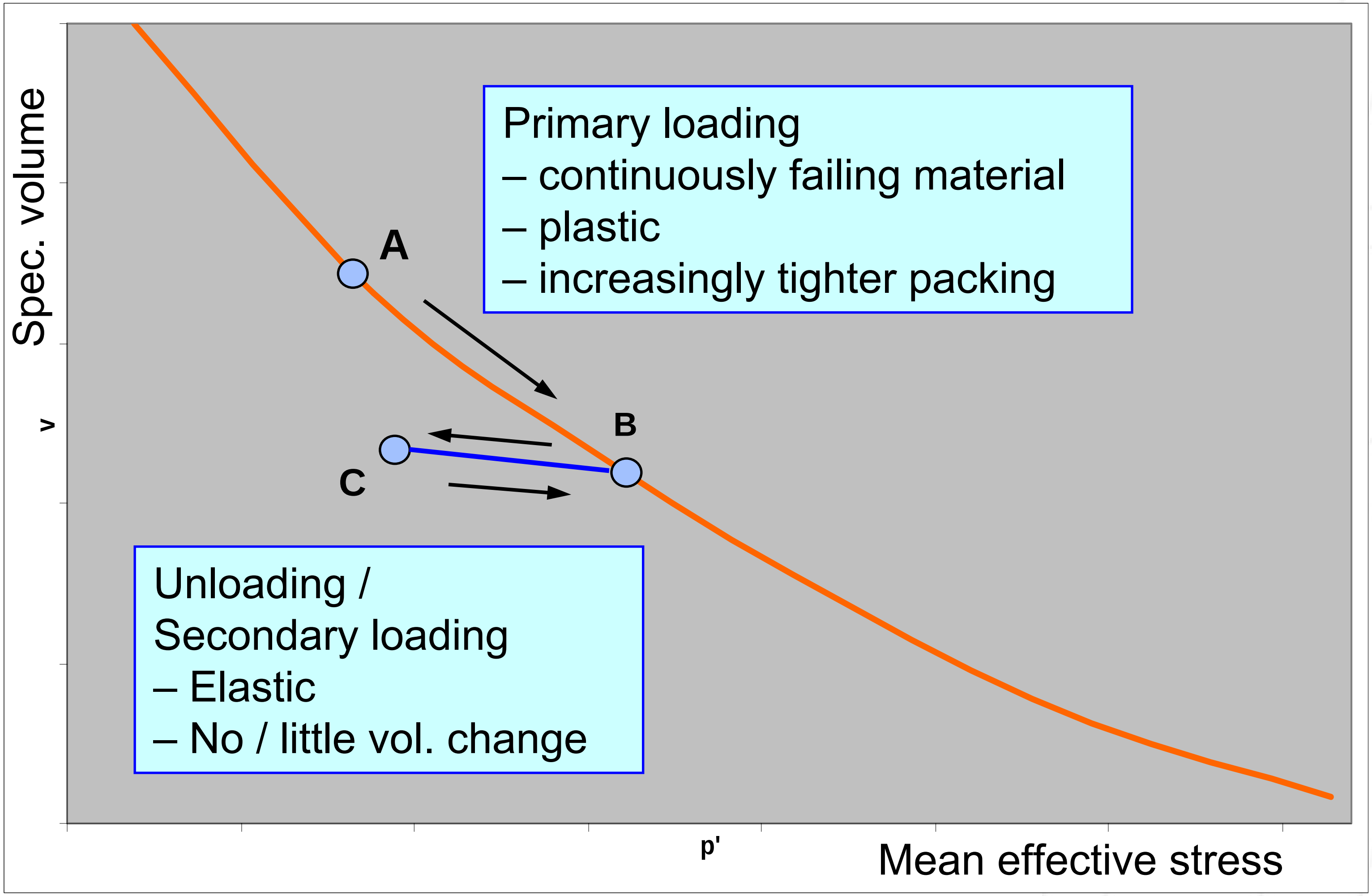
- At pore level, continuous pore wall failure is taking place during compaction
- At bulk level, compaction will be observed as permanent deformation of pore space (plasticity)
- The soil has *no memory* of its past stress history
  - each packing level can be seen as a “new” material with its own poro-elasto-plastic parameters

Other mechanisms – complicating factors

- During a load increase the soil may fracture instead of tighter packing – seen as a sudden reduction of strength
- The material is not "pure". The void space may be partly filled with bonding agents and / or fine-grained material which may break or dissolve during flooding
- Grain particle corners can break off during reorganization
- Fines can settle in pore space or be transported by flowing fluid
- Presence of shear stress may cause dilation in place of or in addition to compaction

These effects are not a part of the grain pack model, but should be considered separately. However they do not weaken subsequent conclusions.

Characteristics of Grain Pack Model



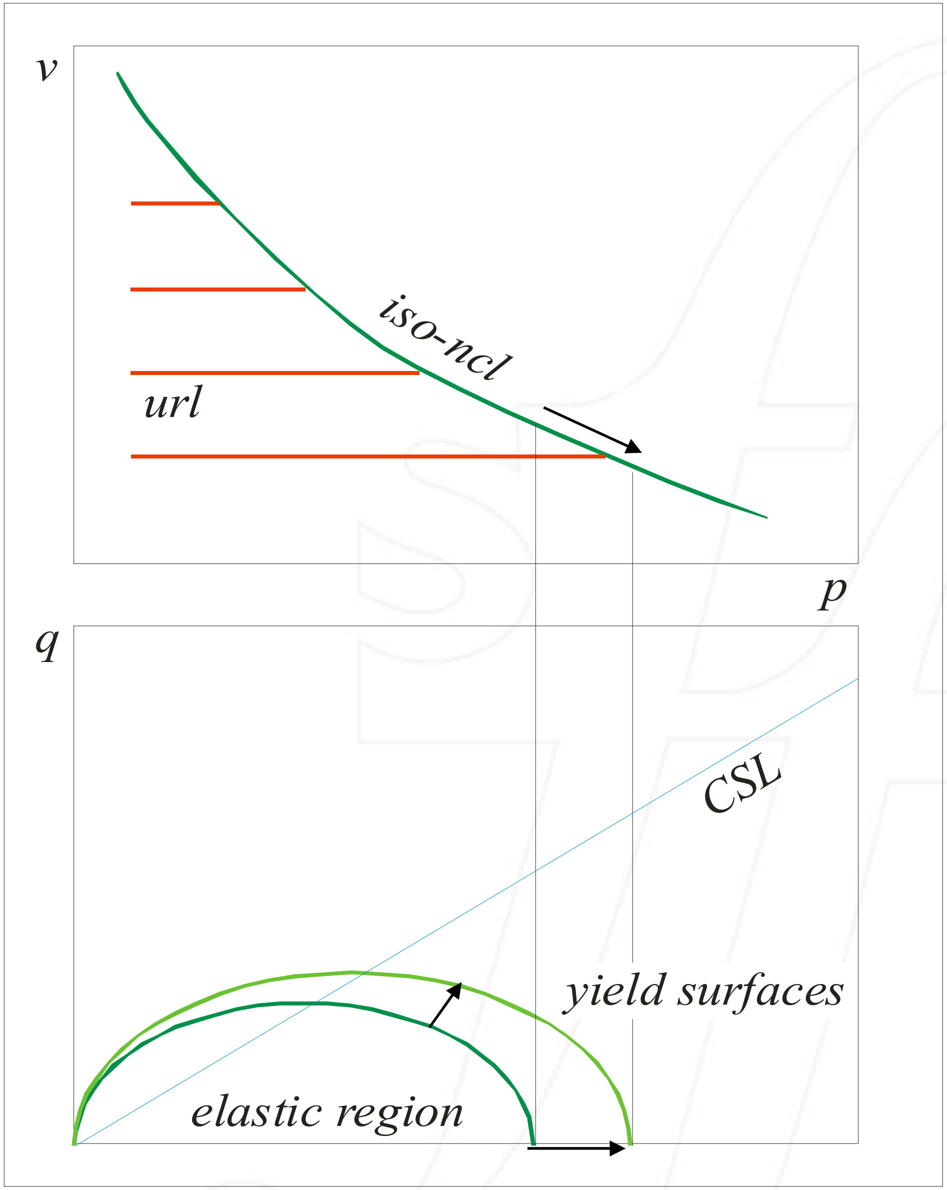
Hence, the grain pack model behaves according to Critical State Theory:

Change of *v* along iso-ncl (isotropic normal compression line) is determined by expansion of yield surface in the stress plane

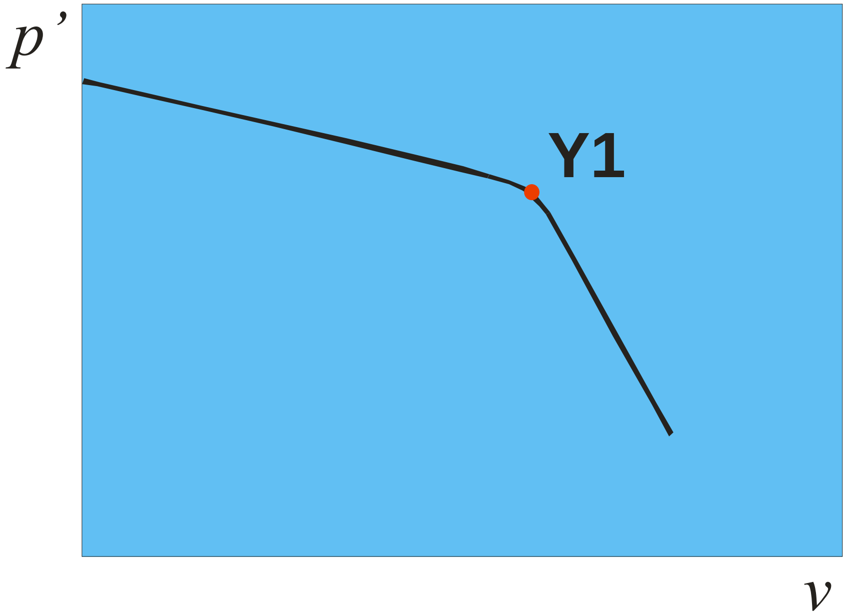
Movement along url's (unloading-reloading lines) occurs in the elastic region in the stress plane

Yield surface expansion is determined by the hardening rule

$$\frac{\partial p}{\partial \varepsilon_p} = p\,\frac{v}{v_0}\,H$$



Experimental determination of yield point

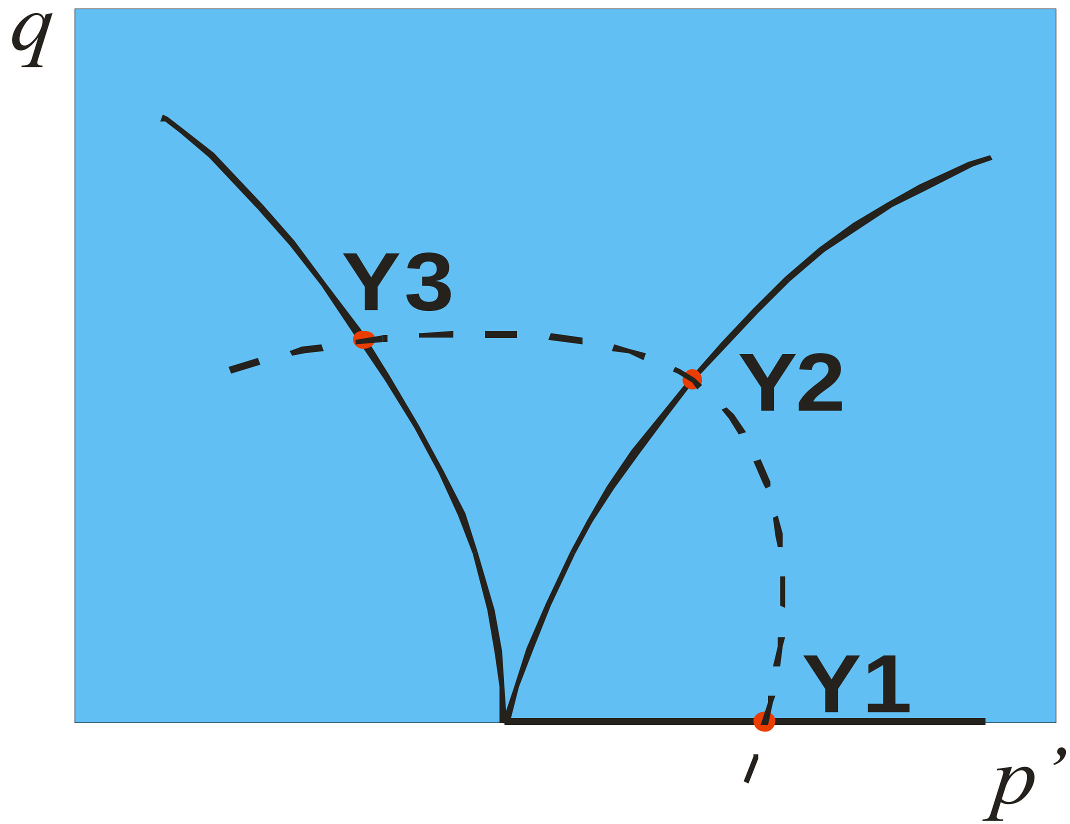


Experiment can e.g. be performed on three *identical* samples, with differing conditions

1) Increasing p_f, σ constant
2) Drained compression
3) Undrained compression

Will provide three different yield points, all indicative of $(p' : q)$ combinations where material will yield.

Yield surface



The $p' : q$ – paths of experiments have been plotted in $p' : q$ – plane, and measured yield points *Y1*, *Y2* and *Y3*.

These three points indicate a yield curve for the material (dashed)

In $p' : q$ – space we would have a yield surface.

The yield surface is a boundary for elastically attainable states.

- **For sands / sandstones, Critical State Theory is *the* appropriate failure model to use.**
  - Not e.g. Mohr-Coulomb (the most popular choice).
  - Definitely not linear elastic

  - In practice we use the special case: Cam Clay Model

Cam clay model

- iso-ncl is a straight line in $\log(p)\!:\!v$ plane
- Yield surfaces are ellipses
  - Horizontal axis always present value of p
  - Vertical axis determined by critical state angle, which can be determined from friction angle

$$v = v_\lambda - \lambda \log p; \quad H = \frac{v_0}{\lambda}$$

The parameters can be determined from specific volume curves,

$$\lambda = \frac{v(p_2) - v(p_1)}{\log(p_2 / p_1)},$$

with p_1 and p_2 chosen such that the iso-ncl fits the data as good as possible

Grain packing – consistent compaction model

Assume compaction in a primary loading process can be described by

$$K(p) = K_0 + a(p - p_0) + b(p - p_0)^2$$

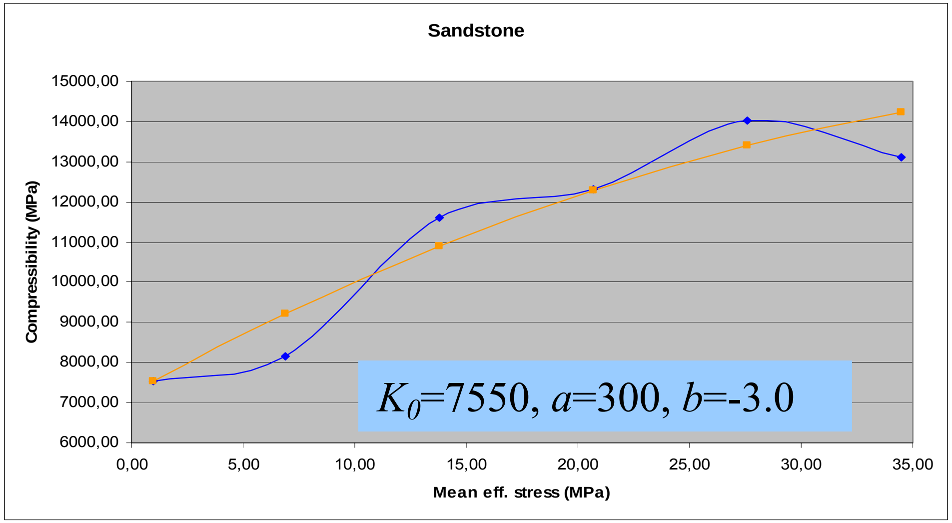
To ensure hardening under compaction we require $a > 0$

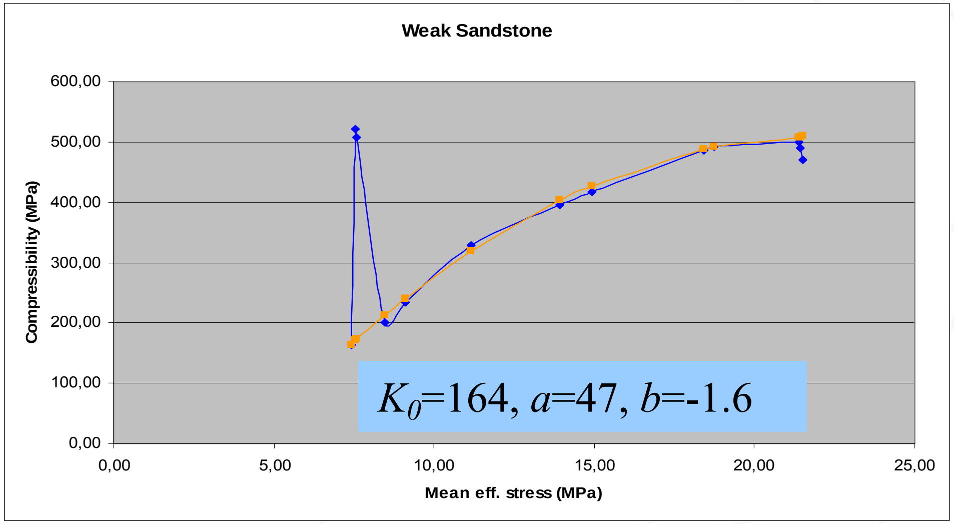
For a pure packing process we would expect $b \geq 0$.
(upwards concave curve, i.e. accelerated hardening)

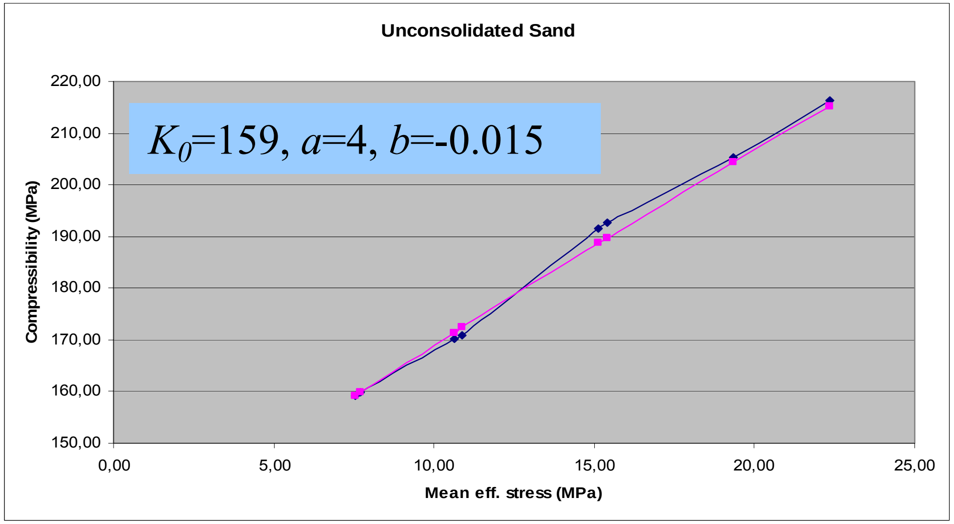
a and b should depend on the initial compressibility K_0, and such that two different compressibility curves satisfy,

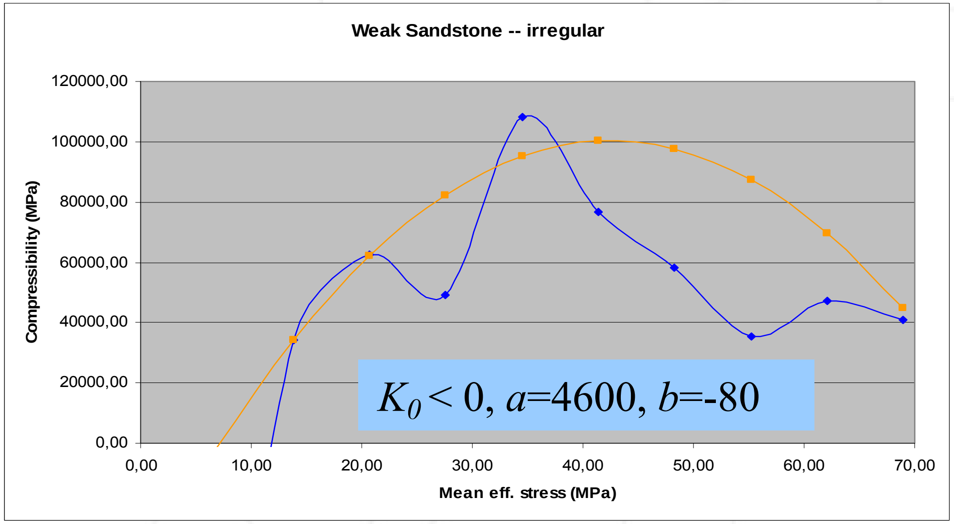
$$K_0^1 < K_0^2 \Rightarrow a_1 < a_2$$

Examples compaction curves; measured and polynomial approximation

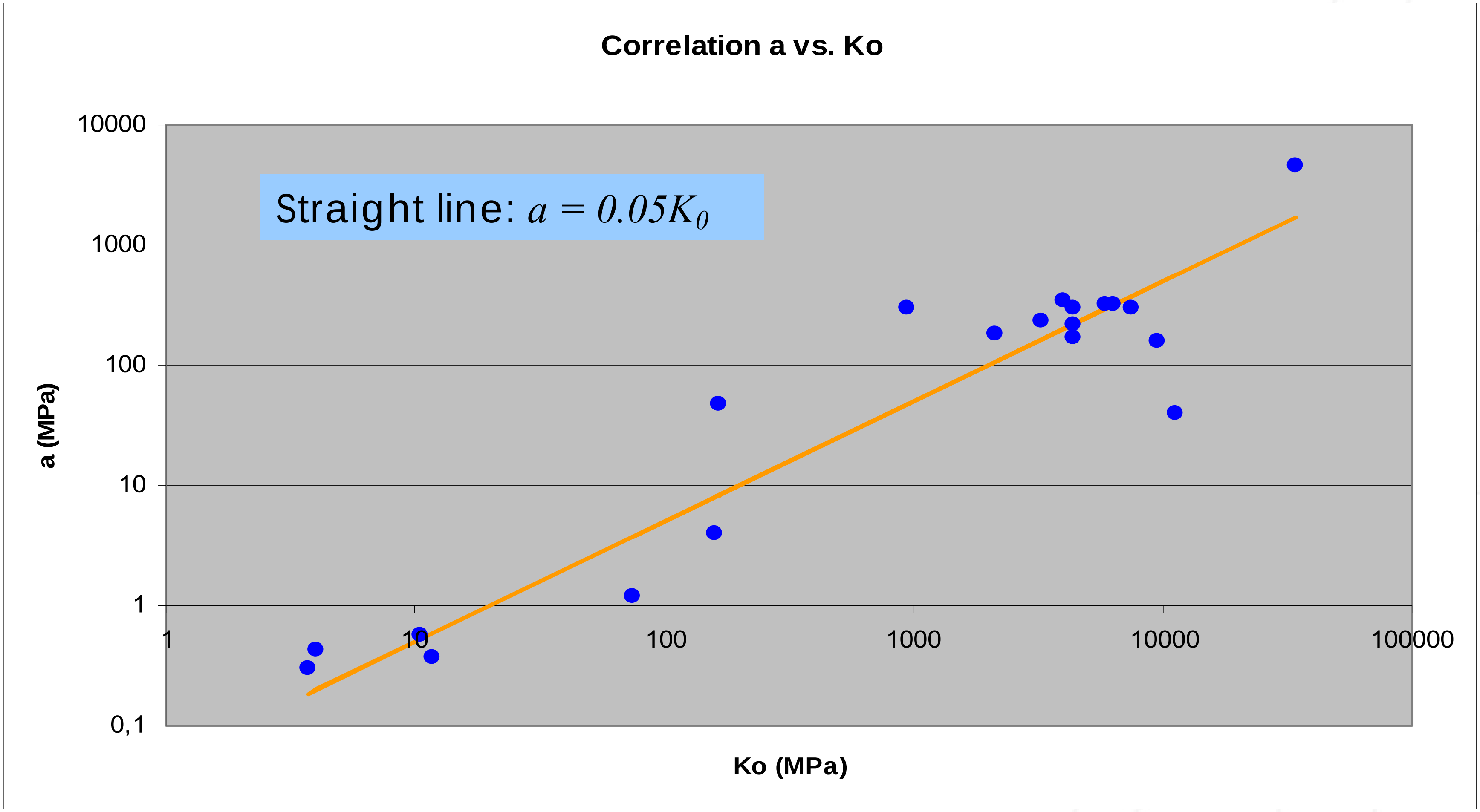




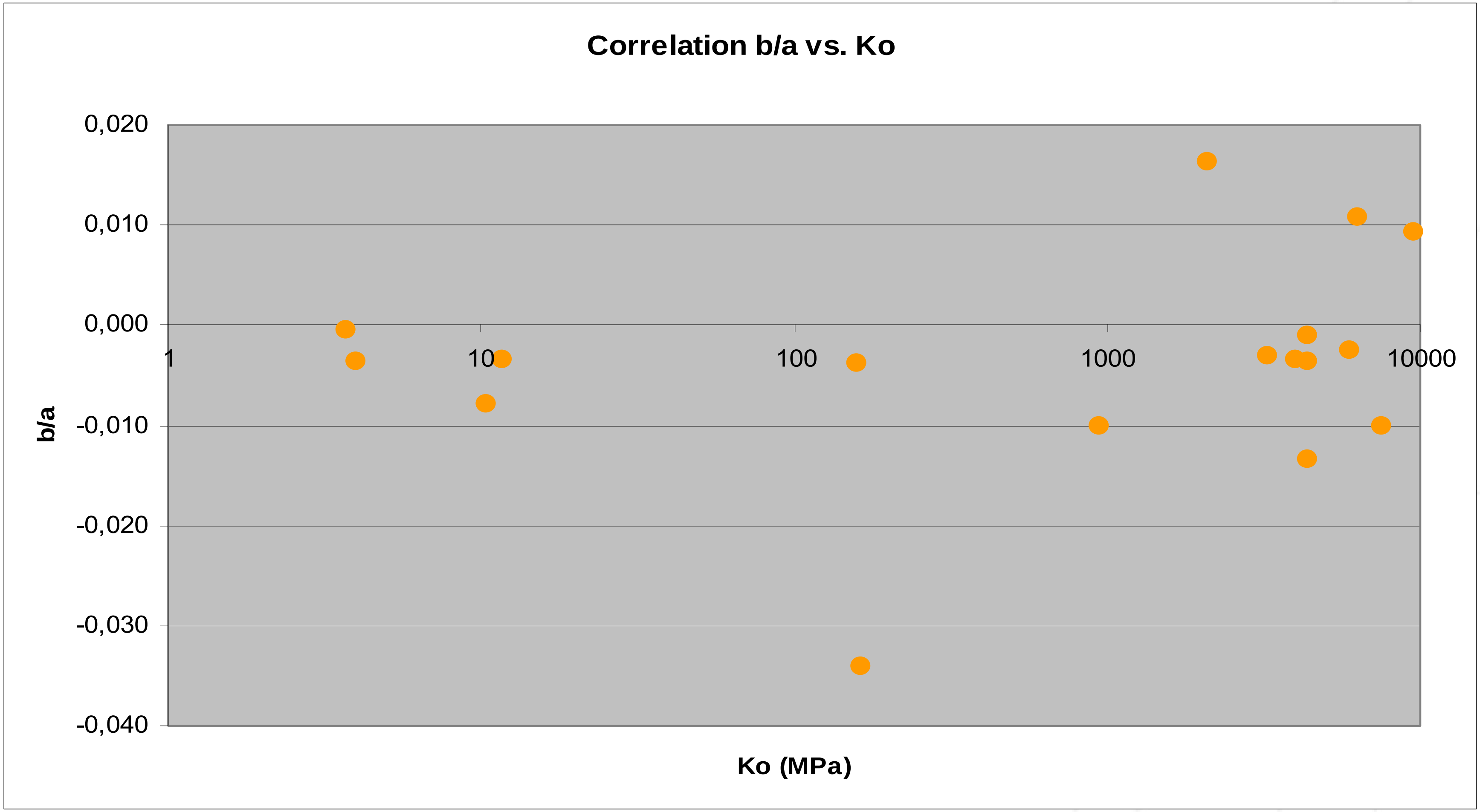




Coefficient a vs. initial compressibility K_0



b/a vs. initial compressibility K_0

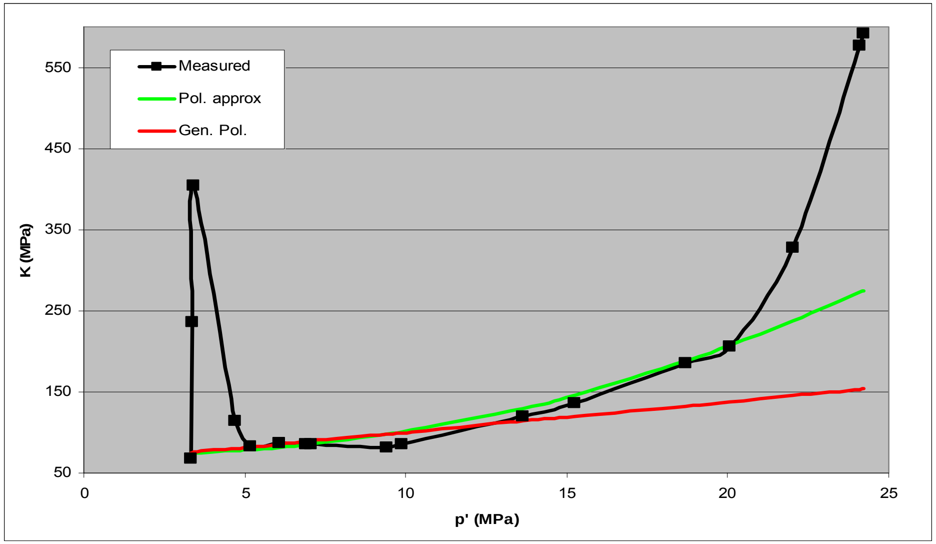


Observations

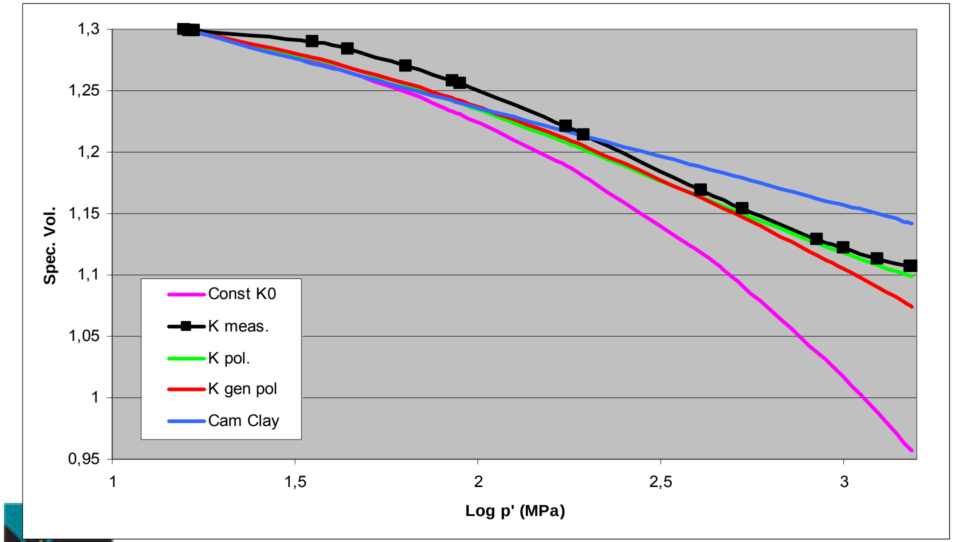
based on soil samples from six North Sea sandstone reservoirs

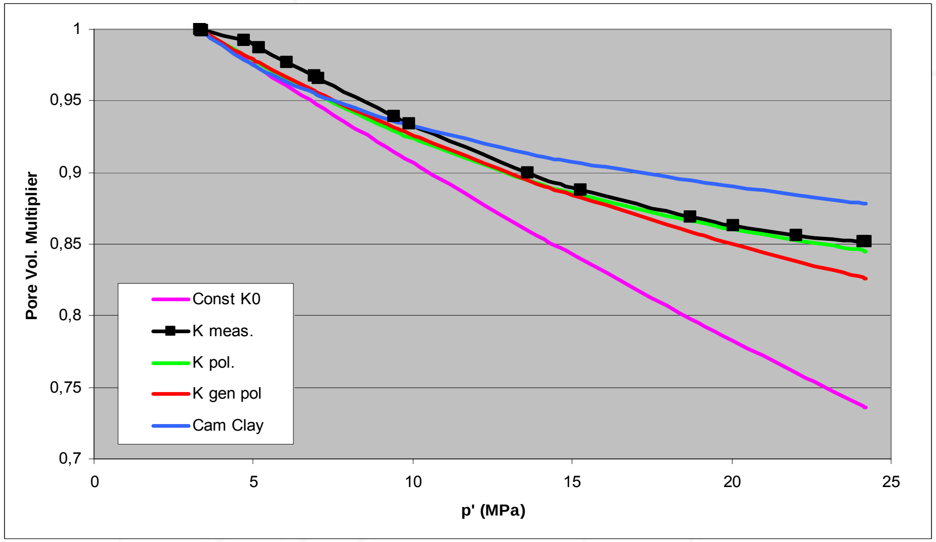
- The polynomial approximation fits most data well
  - easier determination of λ and H.
- The general approximation fits to varying degree, and should only be used when no measured data exist
- Variation of specific volume with mean eff. stress:
  - The widely used constant compressibility assumption is almost always the worst fit to "correct" curve
  - Cam clay model fits data reasonably well for limited load, but is also often "way off"
    - Could be improved by allowing λ and H to vary with p'
- $v(p')$ should always be modeled as irreversible

Example Unconsolidated Sand

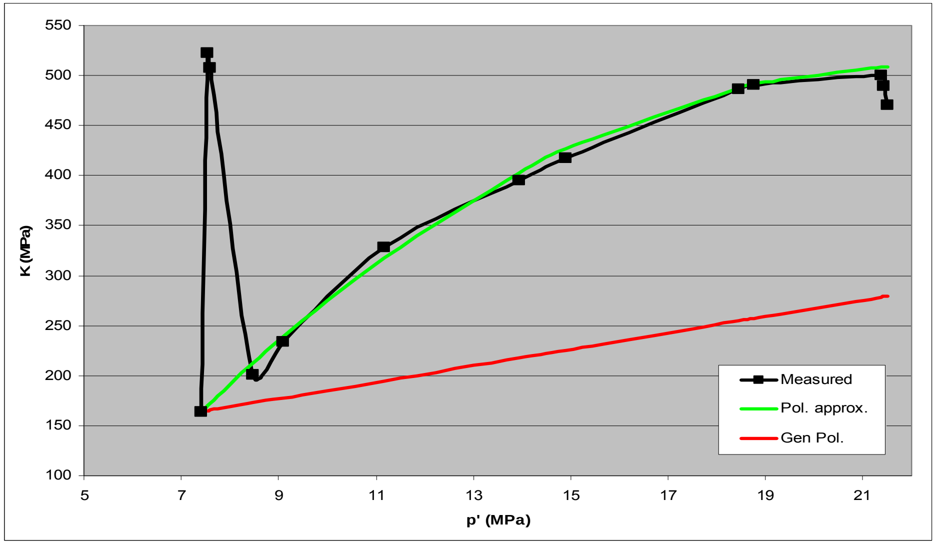


$$a = 1.2$$
$$b = 0.4$$
$$\lambda = 0.08$$
$$H = 16.4$$

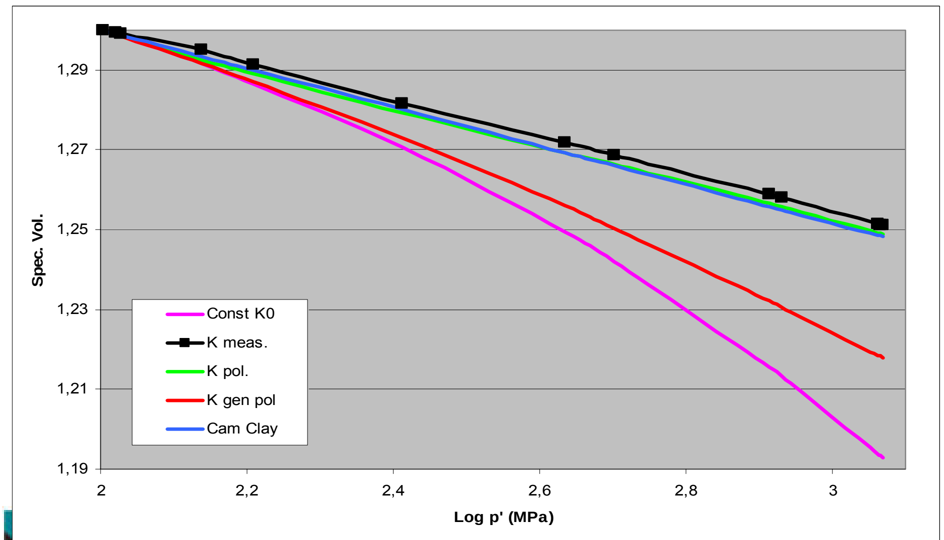


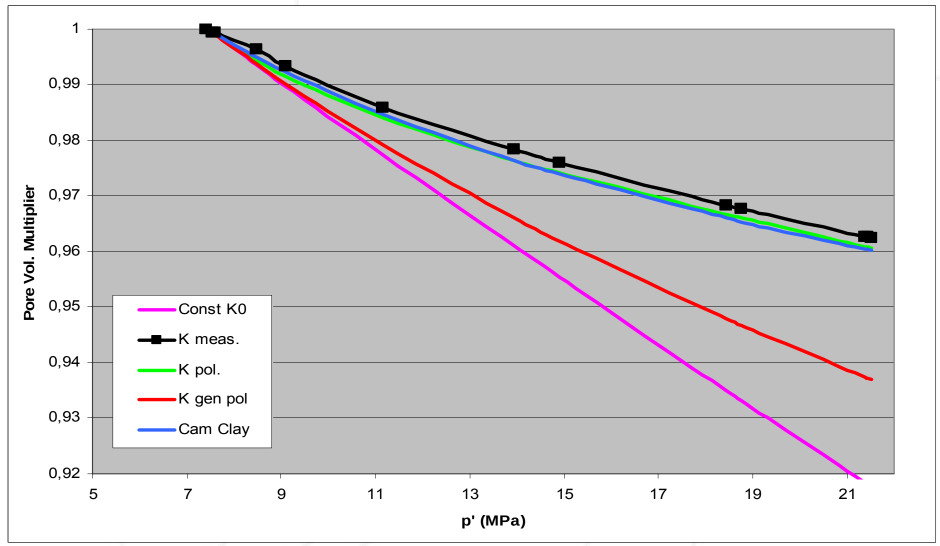


Example Weak Sandstone

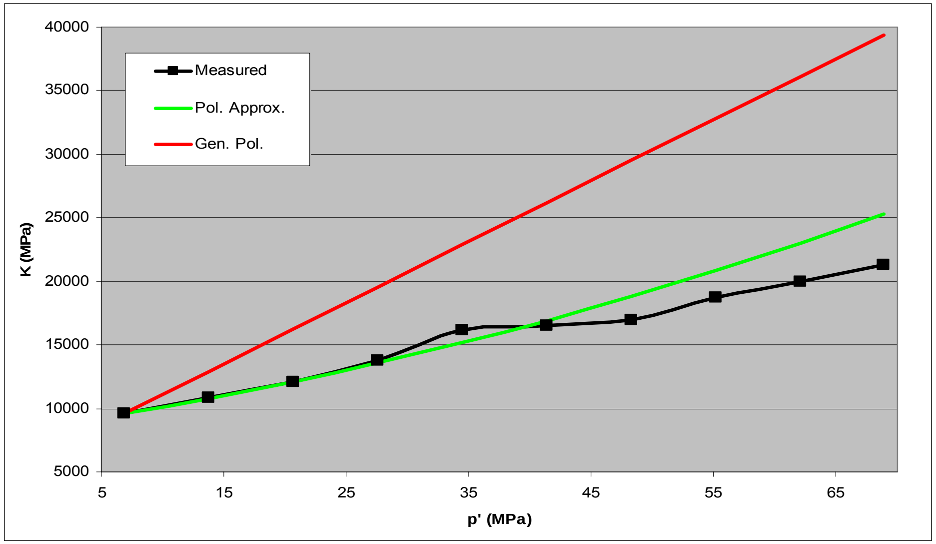


$$a = 47$$
$$b = -1.6$$
$$\lambda = 0.0485$$
$$H = 27$$

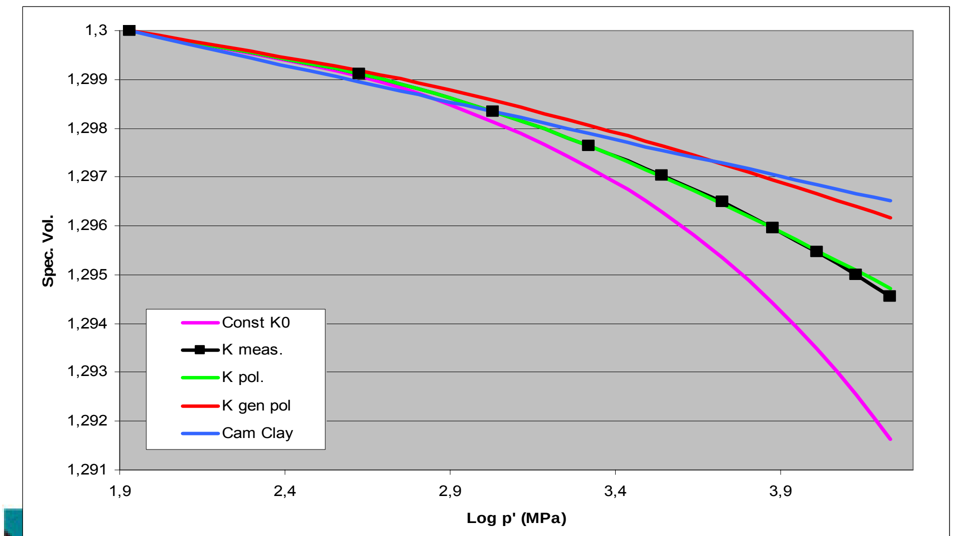


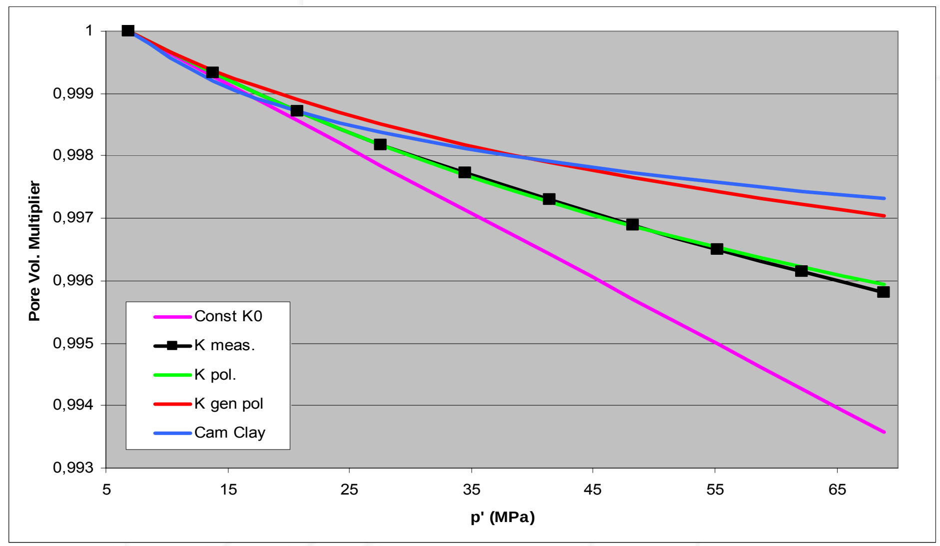


Example medium strength Sandstone



$$a = 160$$
$$b = 1.5$$
$$\lambda = 0.0015$$
$$H = 856$$

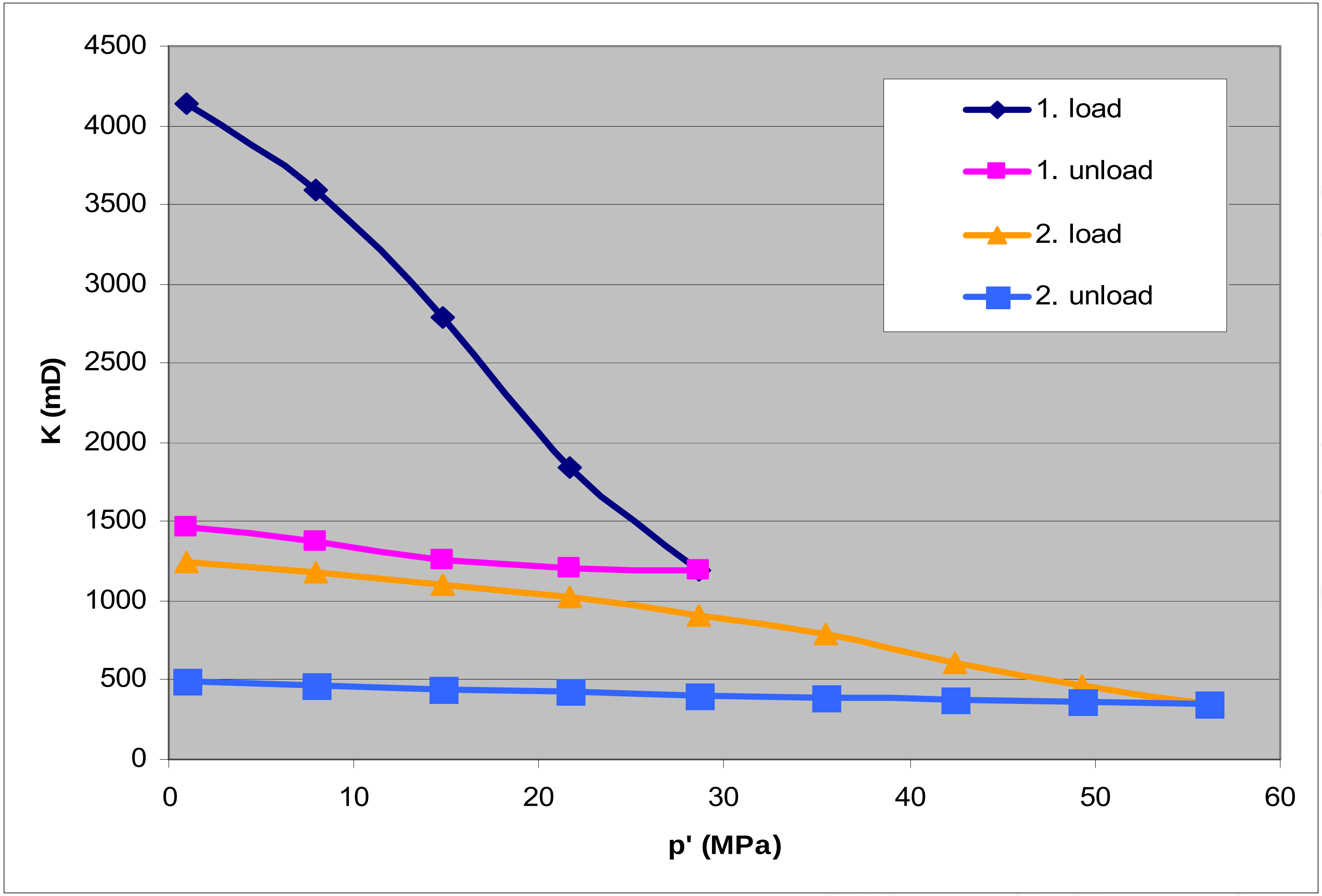




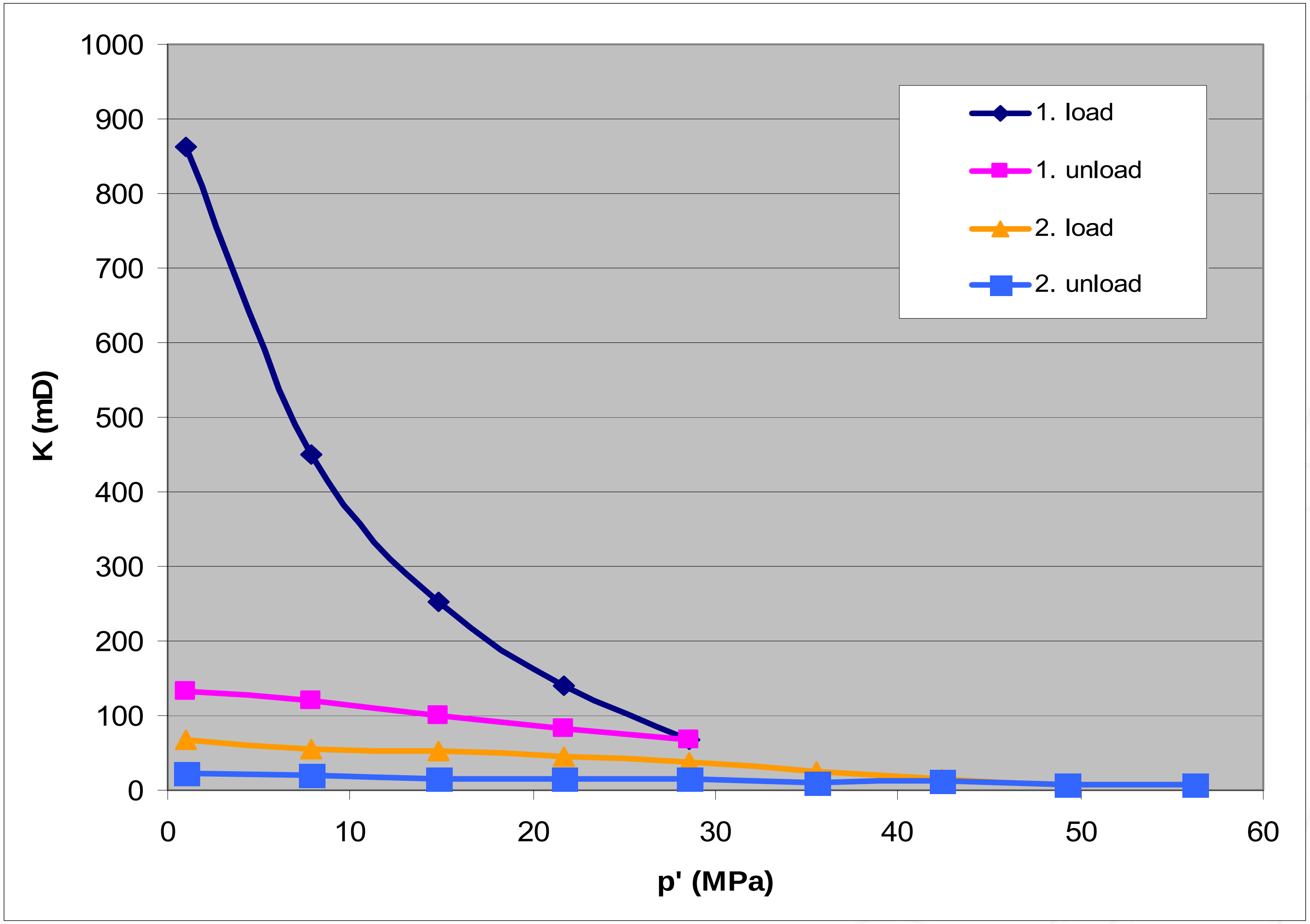
Permeability during compaction

- By the mechanisms of grain packing it is to be expected that
  - Permeability is reduced with loading
  - The decrease will be largest for high initial permeability
  - i.e. heterogeneous soils will tend to be homogenizised by loading

Example lab. test unconsolidated sand

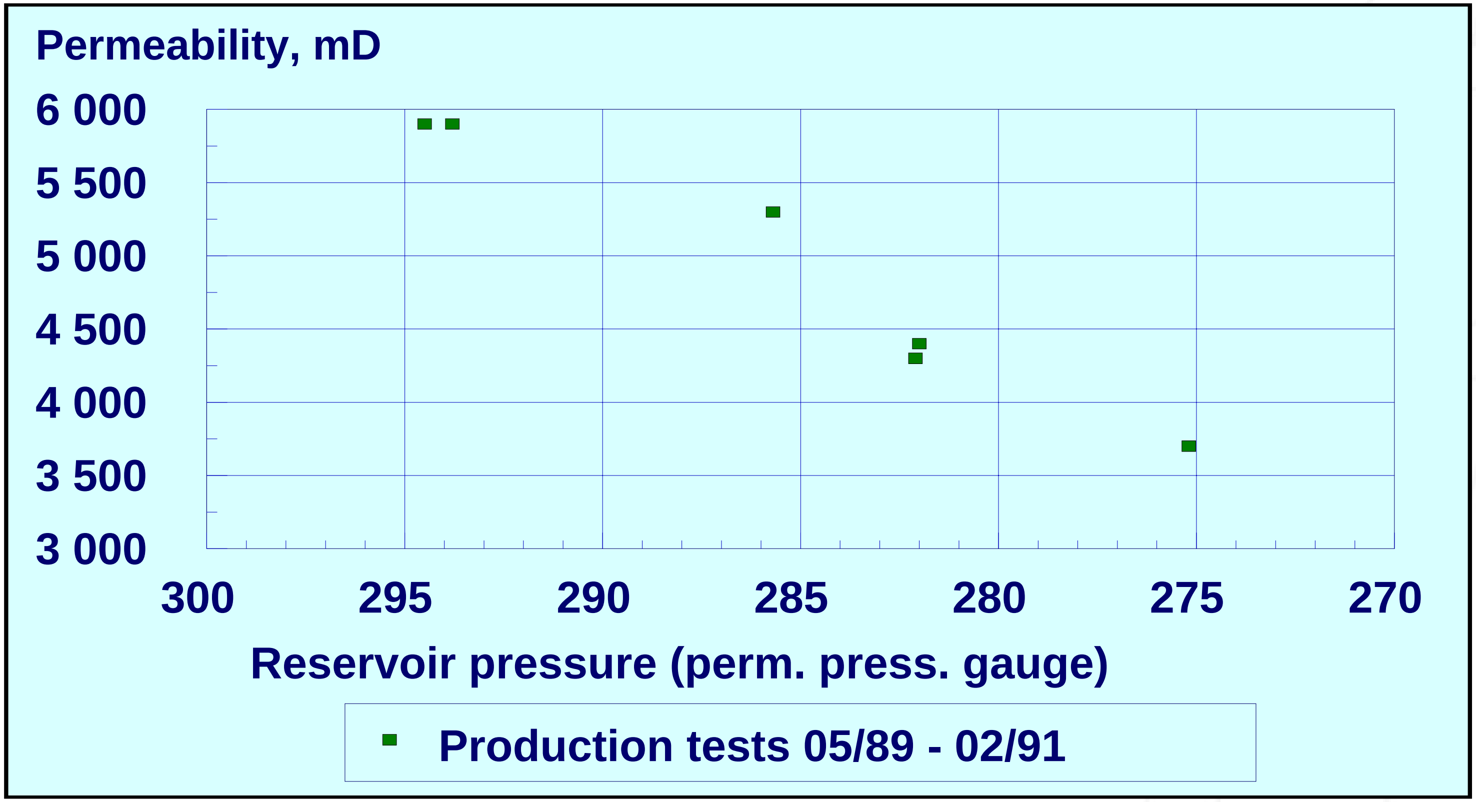


Example lab. test weak sandstone



Field Example weak sandstone

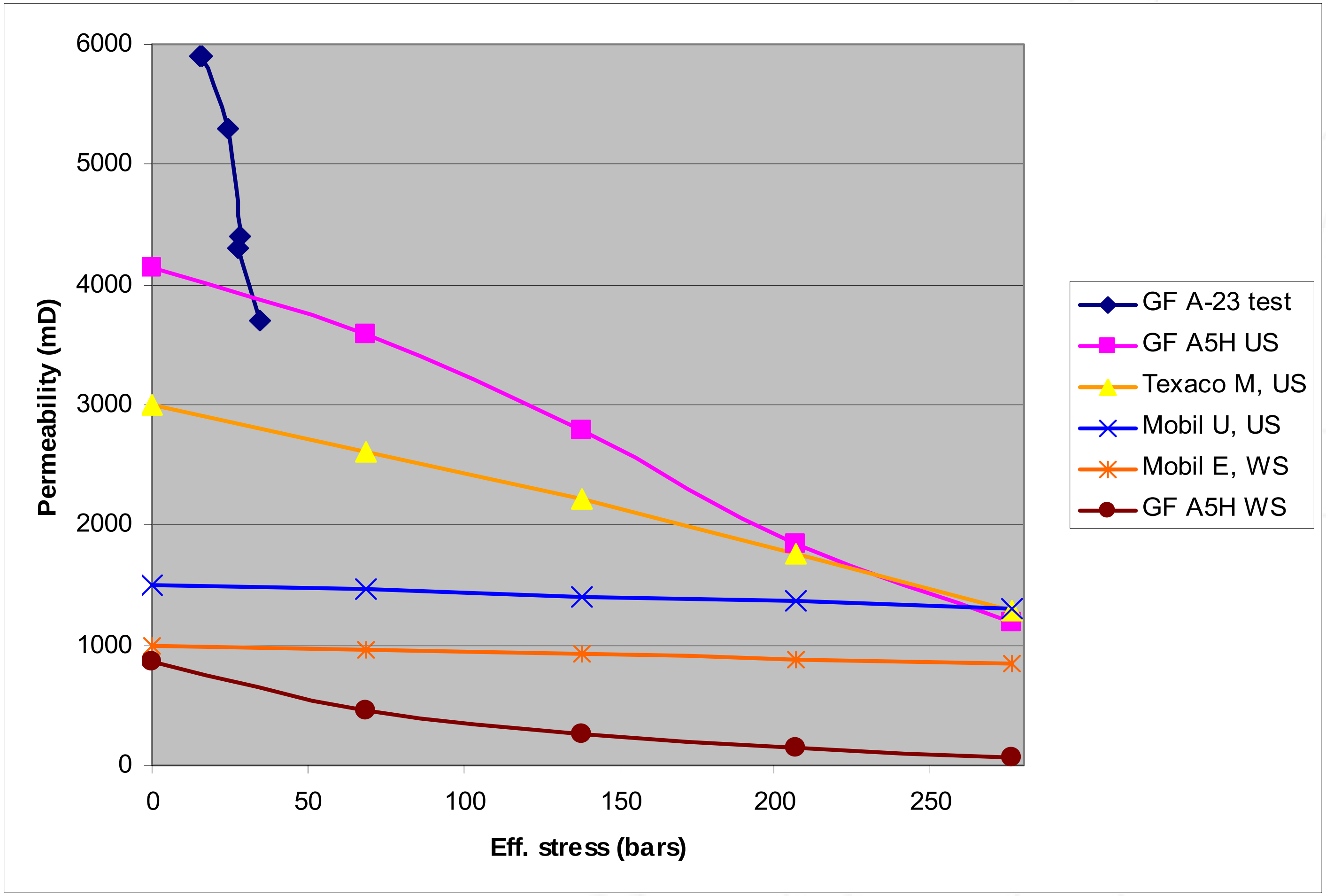
Permeabilities from transient test analysis
Gullfaks Well A-23



Permeability vs. load – measured data

Weak sands appear to have greater loss of conductivity than strong sands

Proposition: Permeability reduction is "proportional to" initial value, i.e. Medium is homogenized by loading



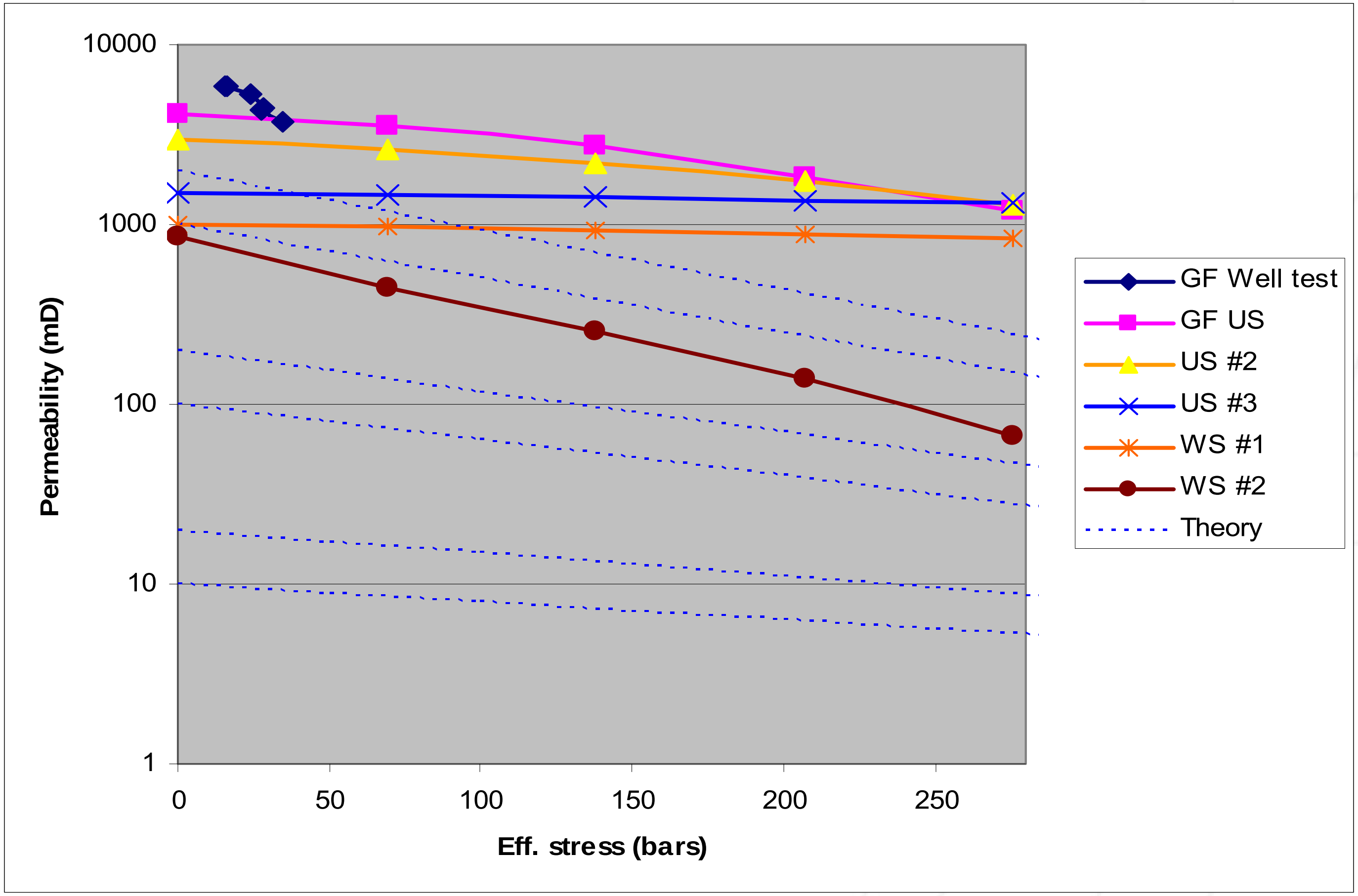
Permeability vs. load – assumption

log(K) linear w. σ,
K(σ*) = 1 mD

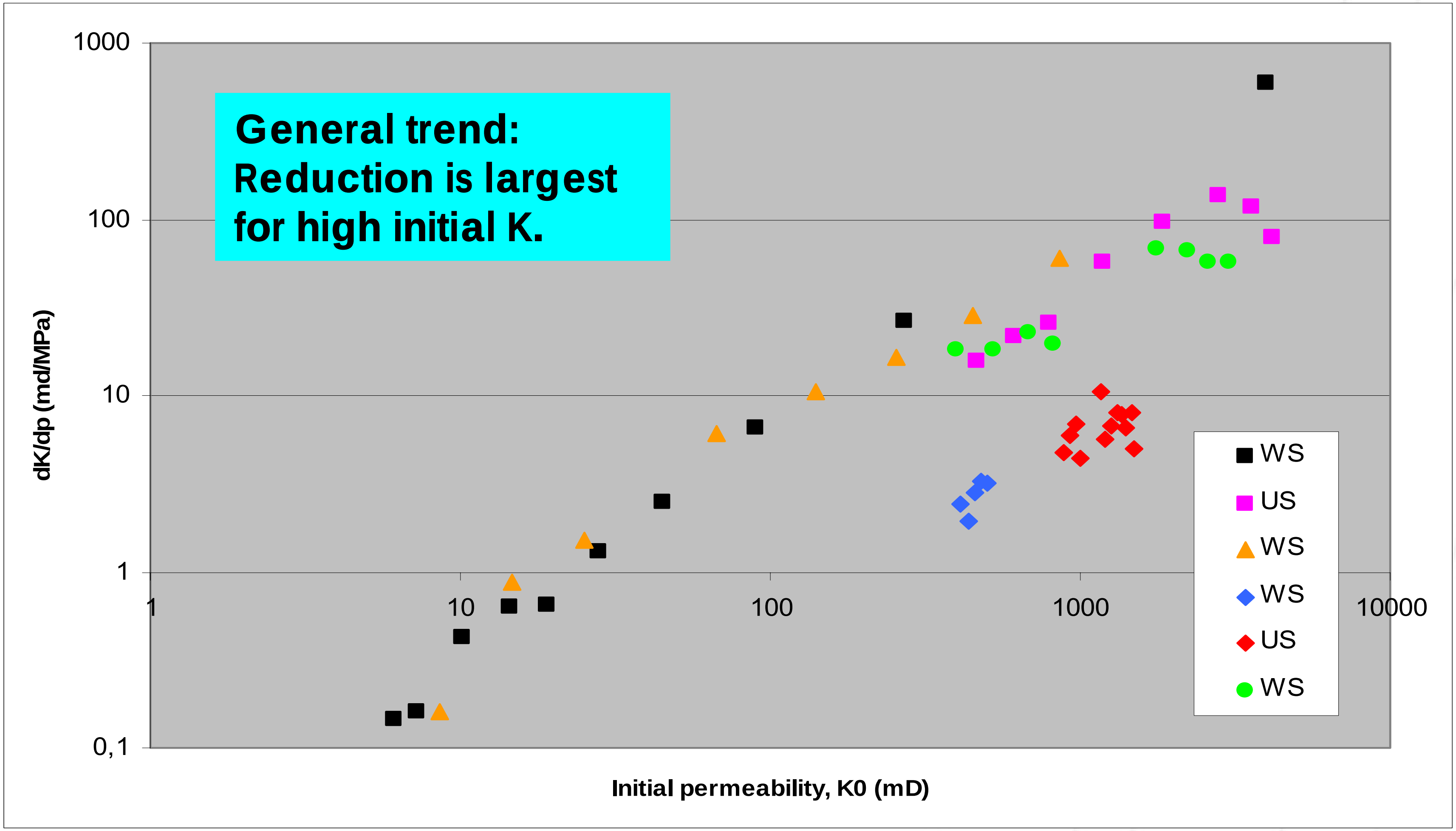
σ* = 1000 bars

fits GF WS

WS: weak sandst.
US: uncons. sand



Permeability *Rate of Change* During Loading



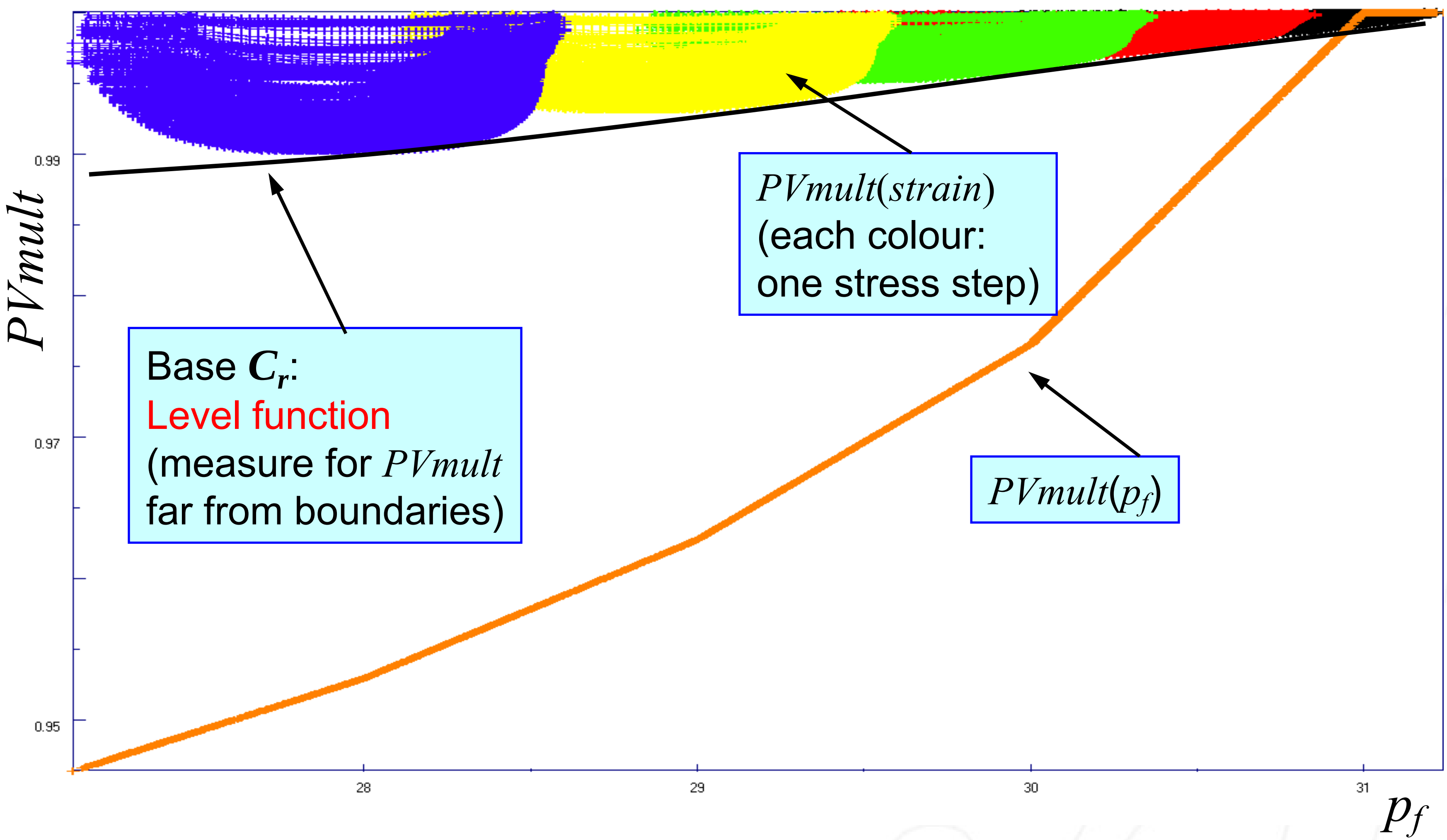
Calculating compaction distribution, Step by step

Rock Mech. Failure Model: Critical State (Cam Clay)

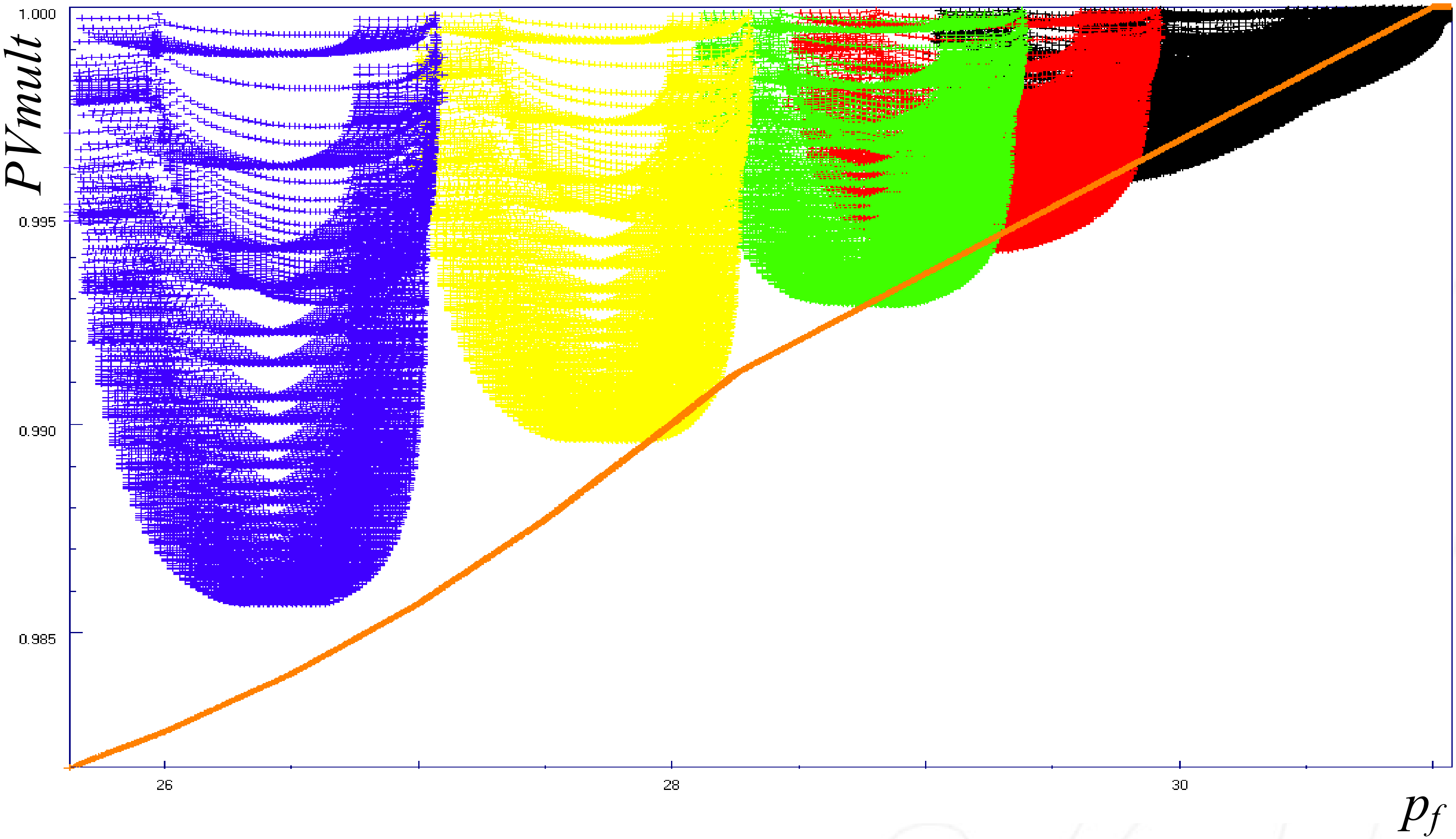
Flow Sim. Compaction function: Derived from Cam Clay

All calculations done on stress step 2; but complete simulation performed at each iteration, for illustrative purposes

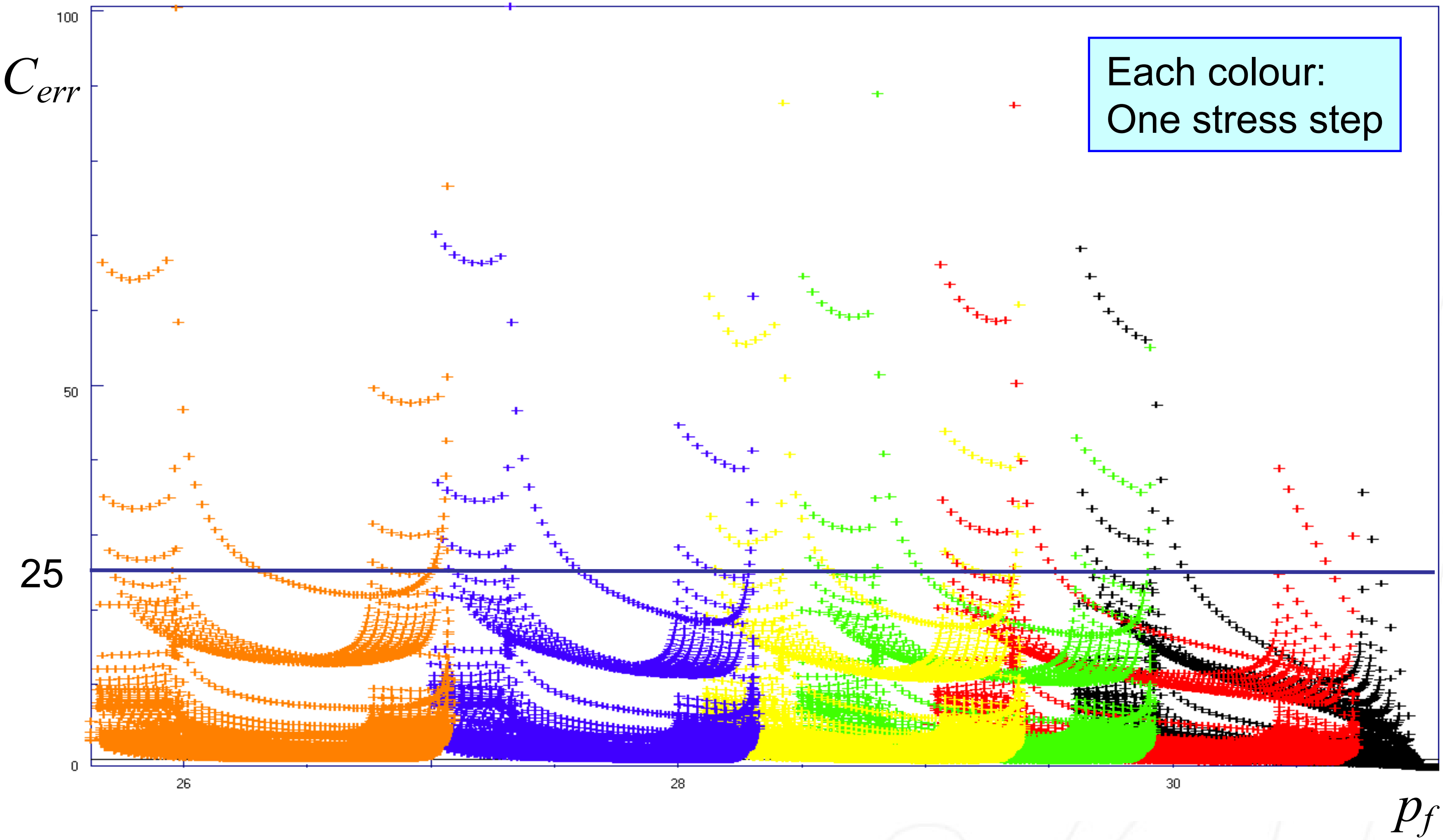
Iteration Step 1: Using $C_r(p_f)$ instead of $C_r(p')$



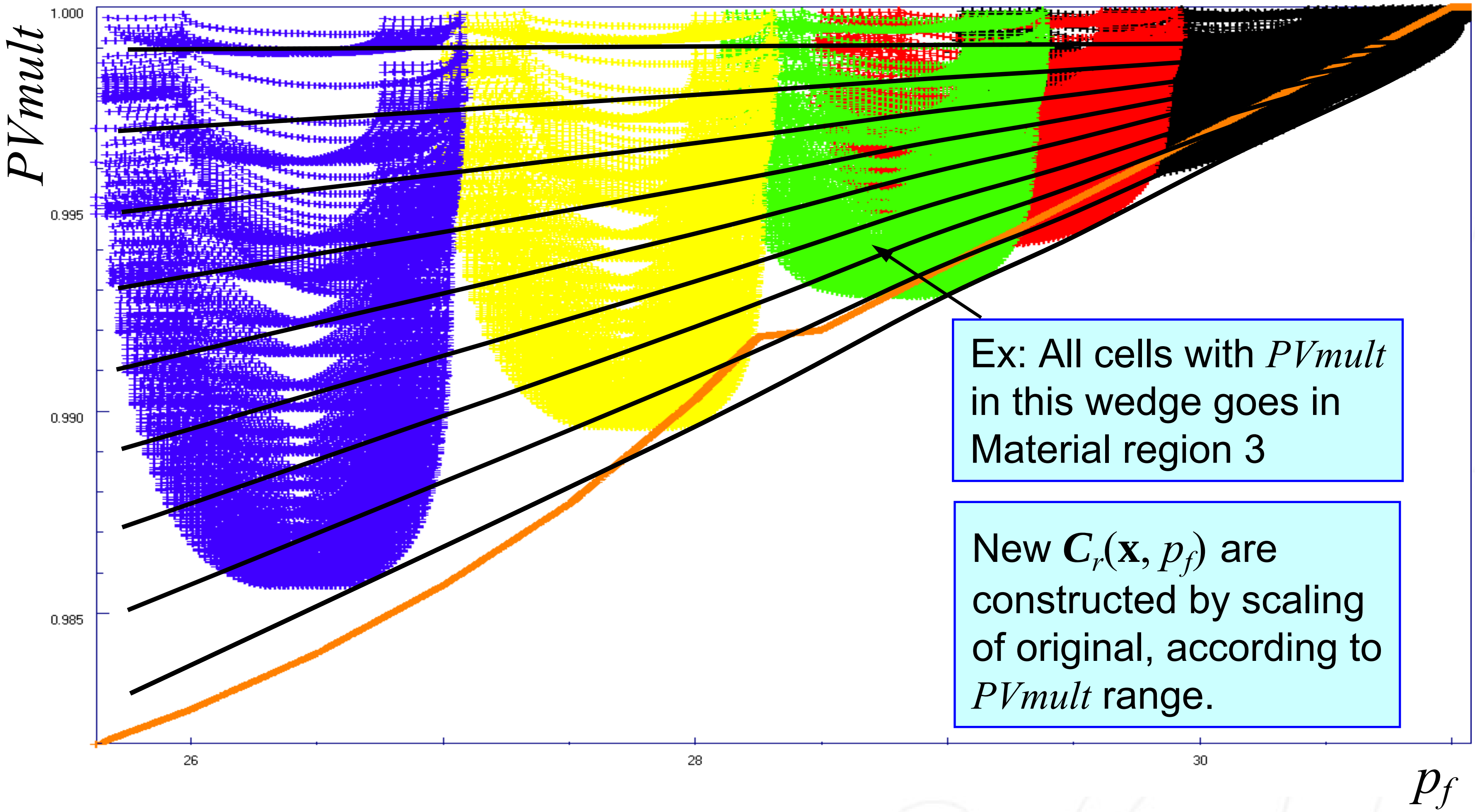
Iteration Step 2: Scale $C_r(p_f)$ to Level Function



Error in Compaction Computation



Iteration Step 3:
Subdivide Reservoir into new Material Regions

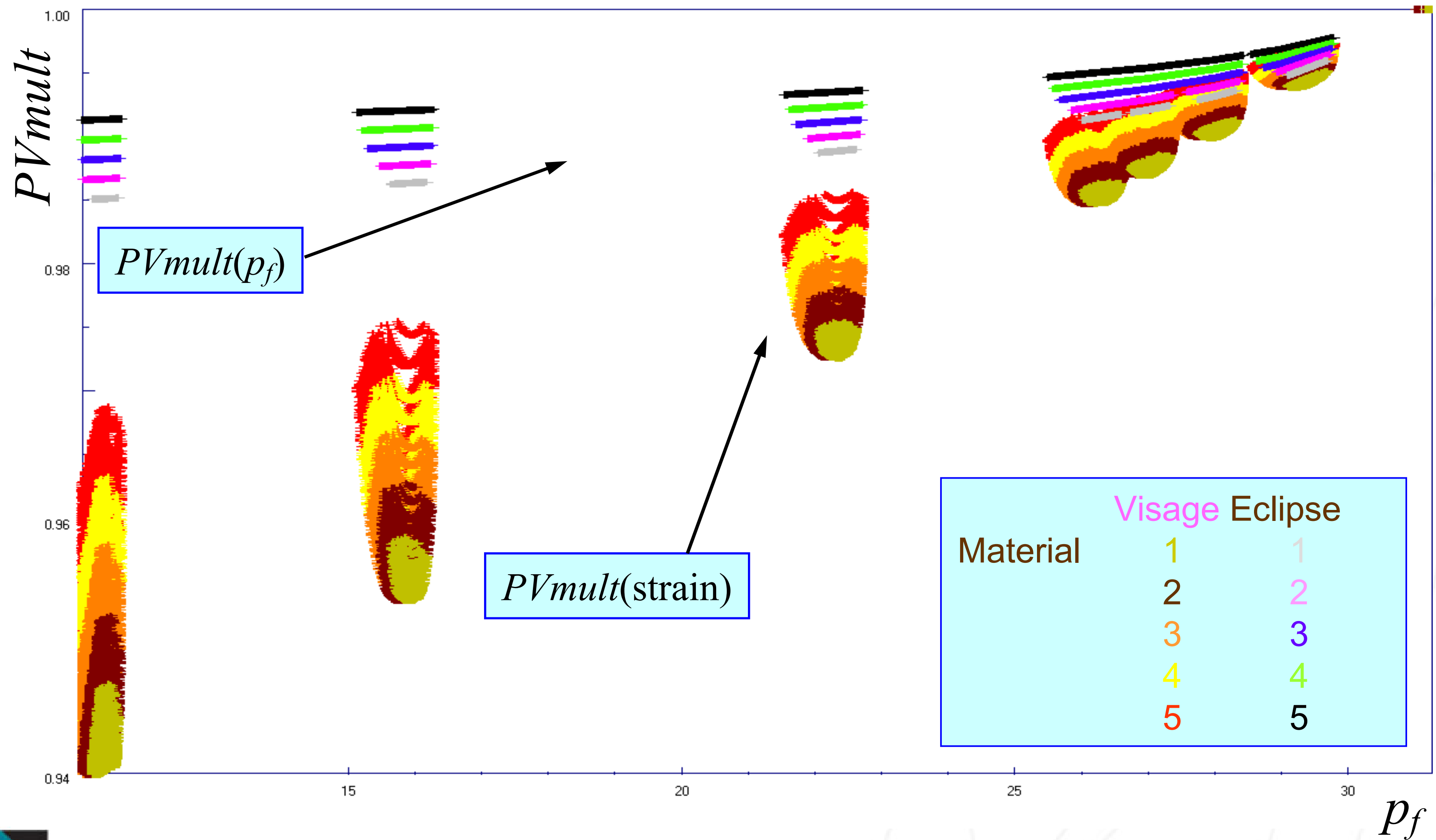


Ex: All cells with $PVmult$ in this wedge goes in Material region 3

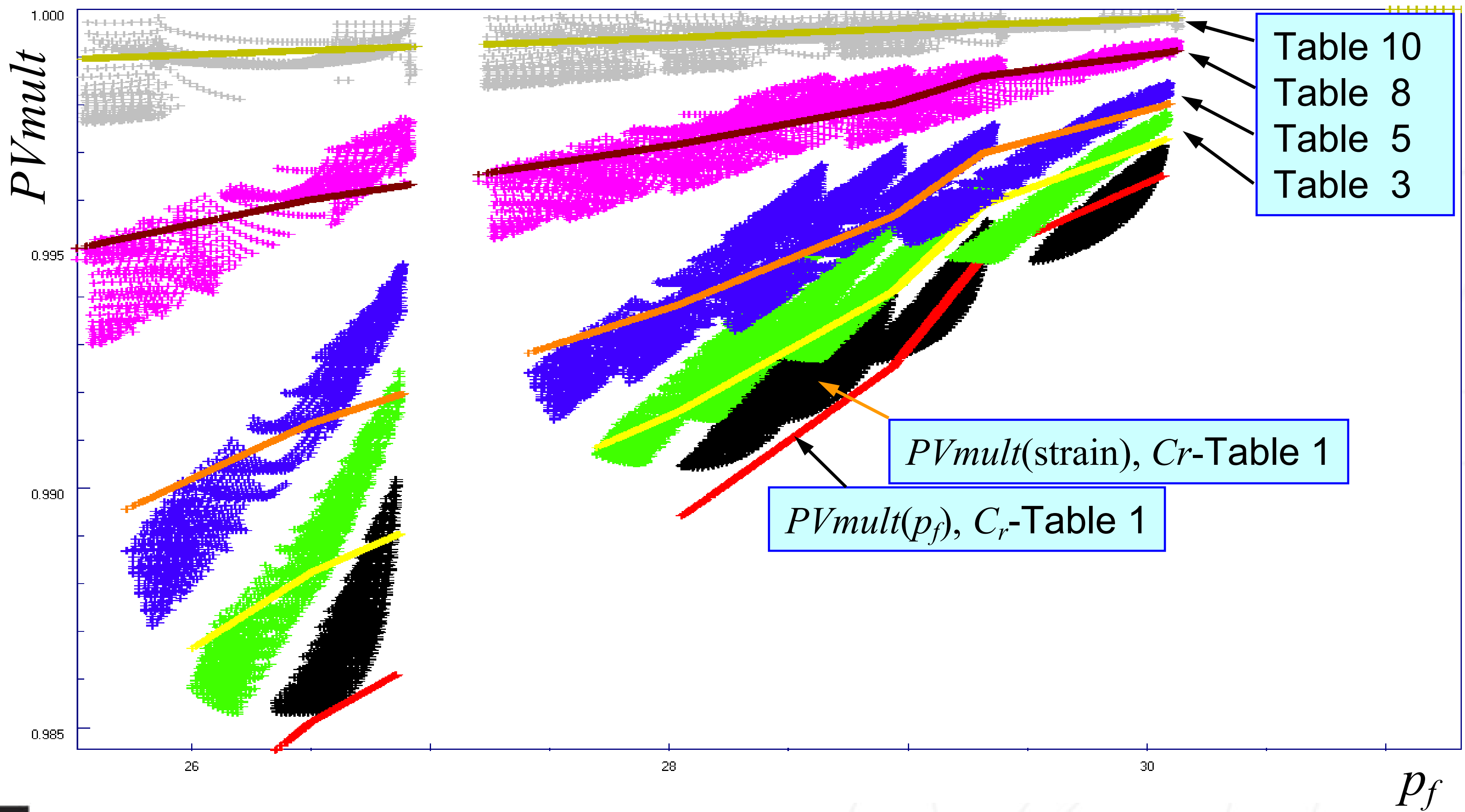
New $\boldsymbol{C}_r(\mathbf{x}, p_f)$ are constructed by scaling of original, according to $PVmult$ range.

New Material Regions, XY View Middle Layer

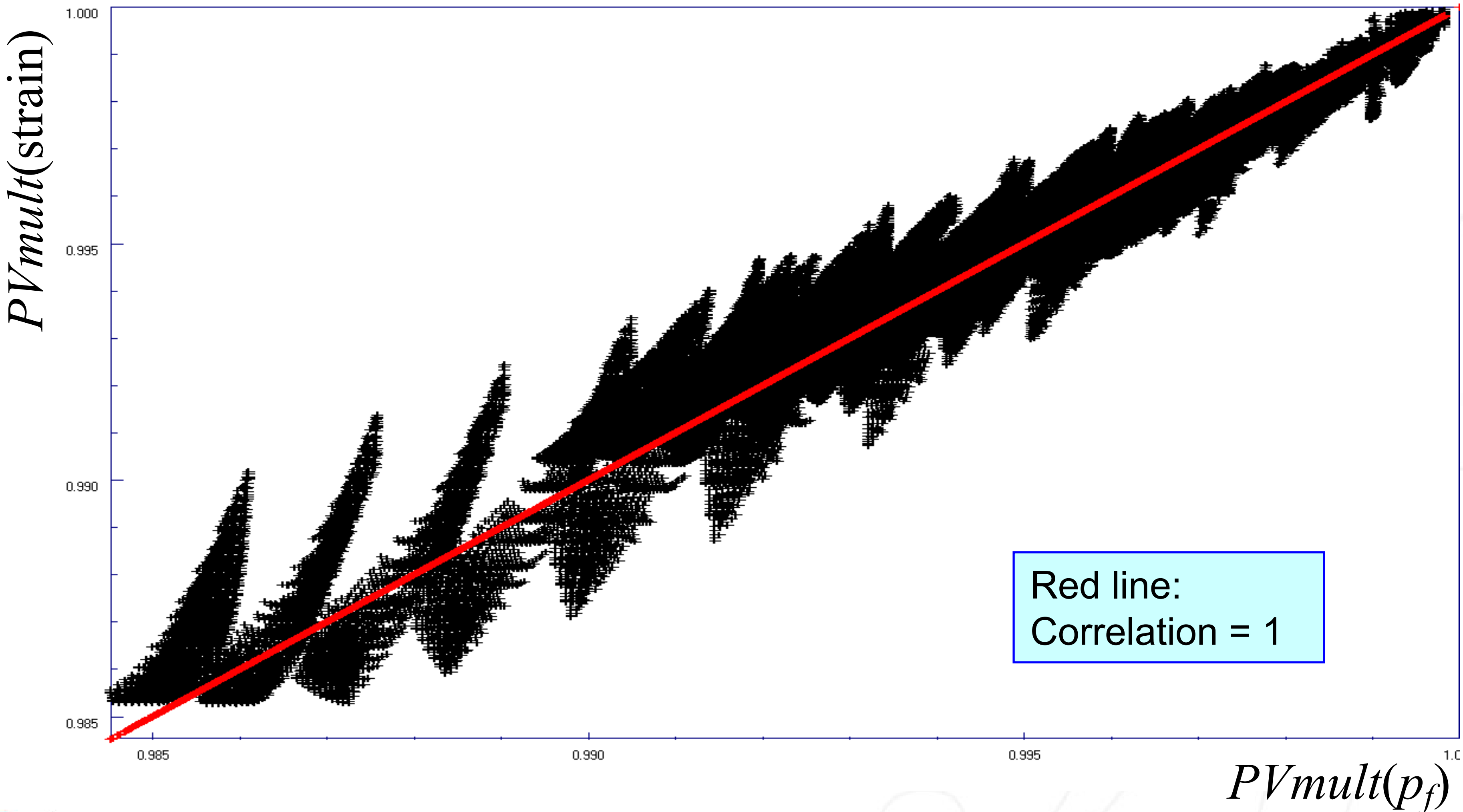
Example Using 10 Material Regions.
Compaction Energy Changed → Level Invalid



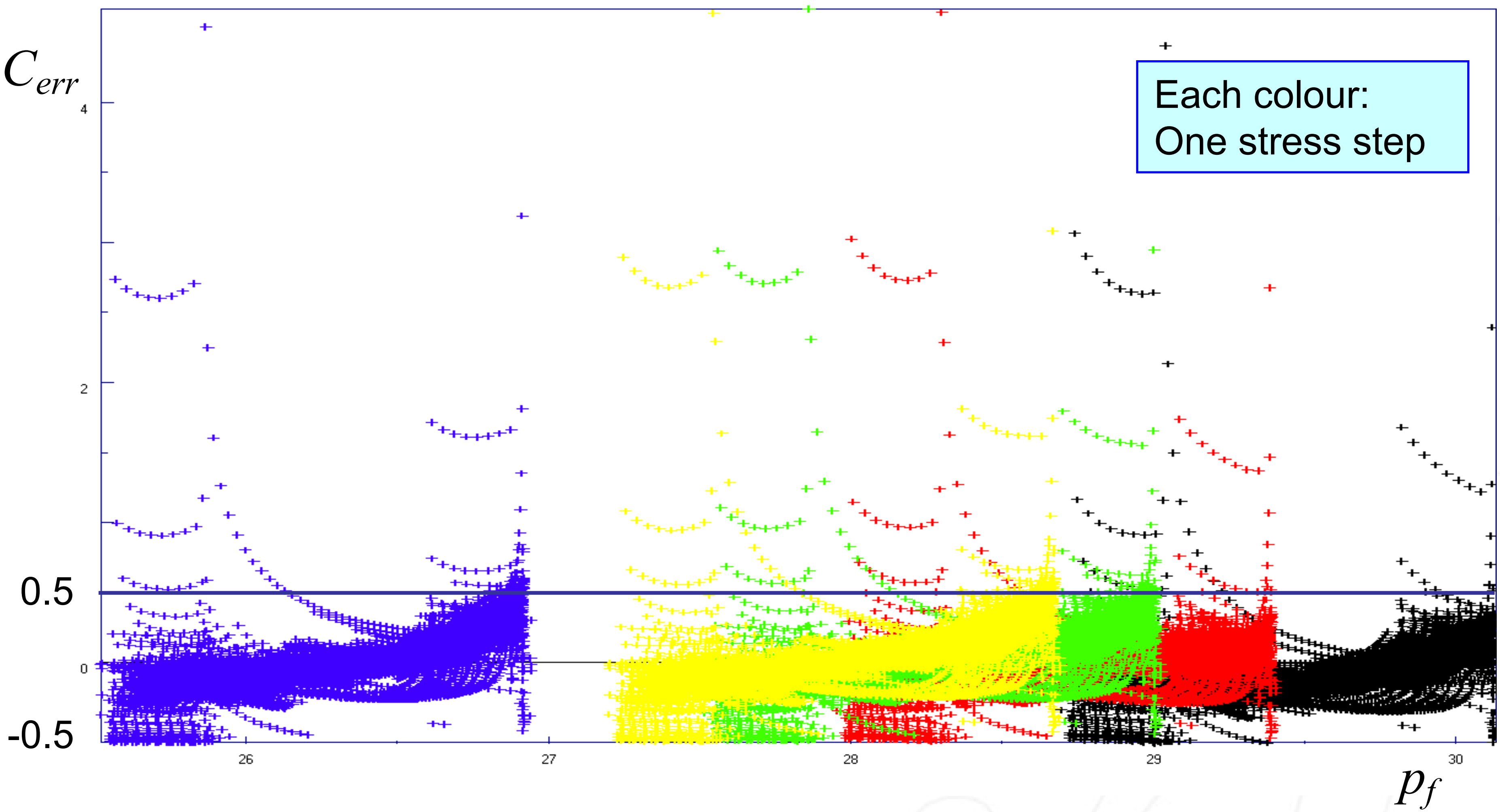
Iteration Step 4:
Adjust Level for all 10 Compaction Functions



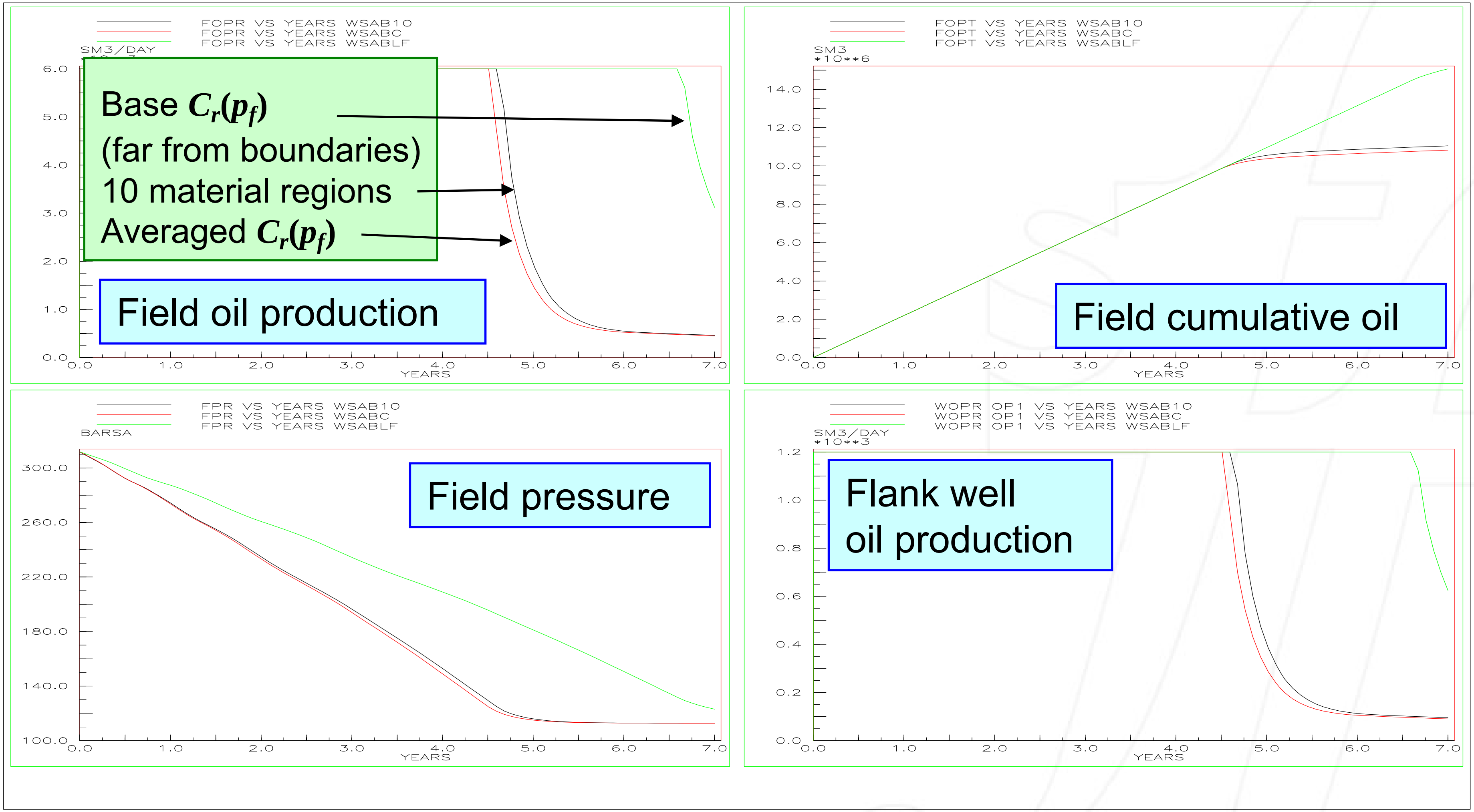
PVmult(strain) vs. *PVmult*(p_f)



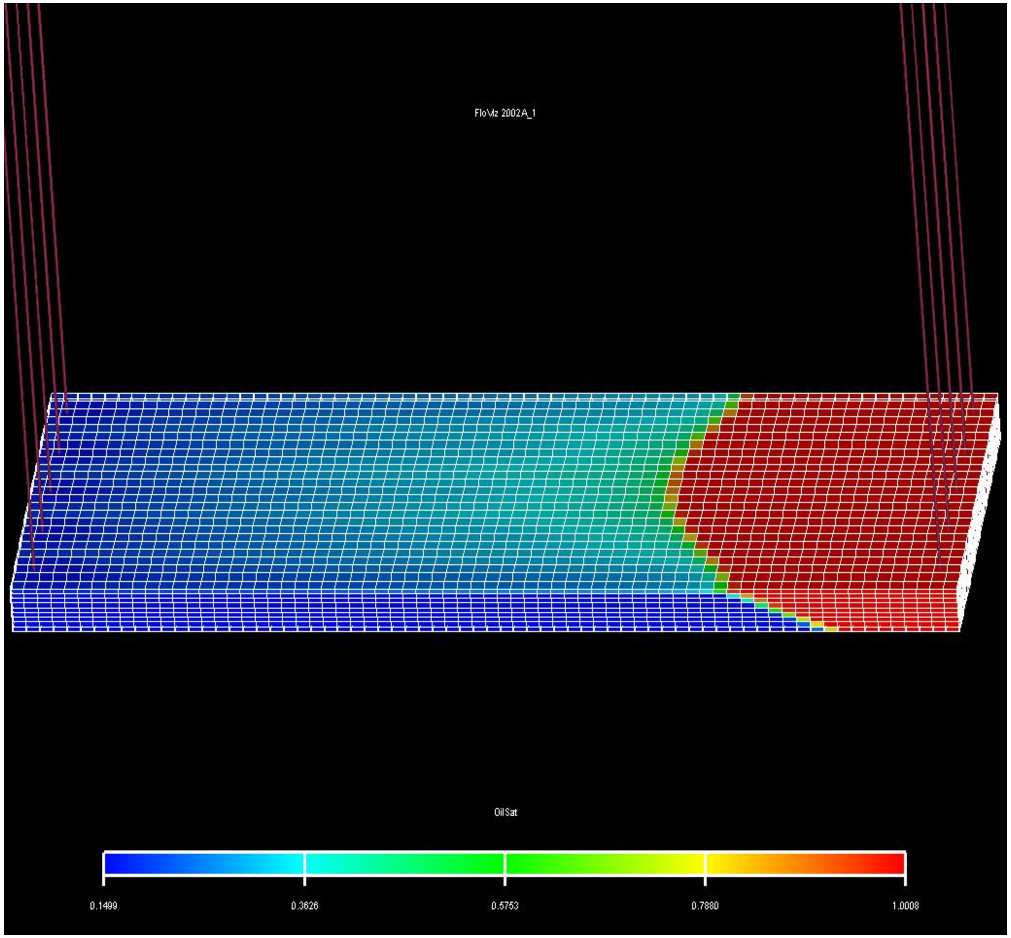
Error in Compaction Computation

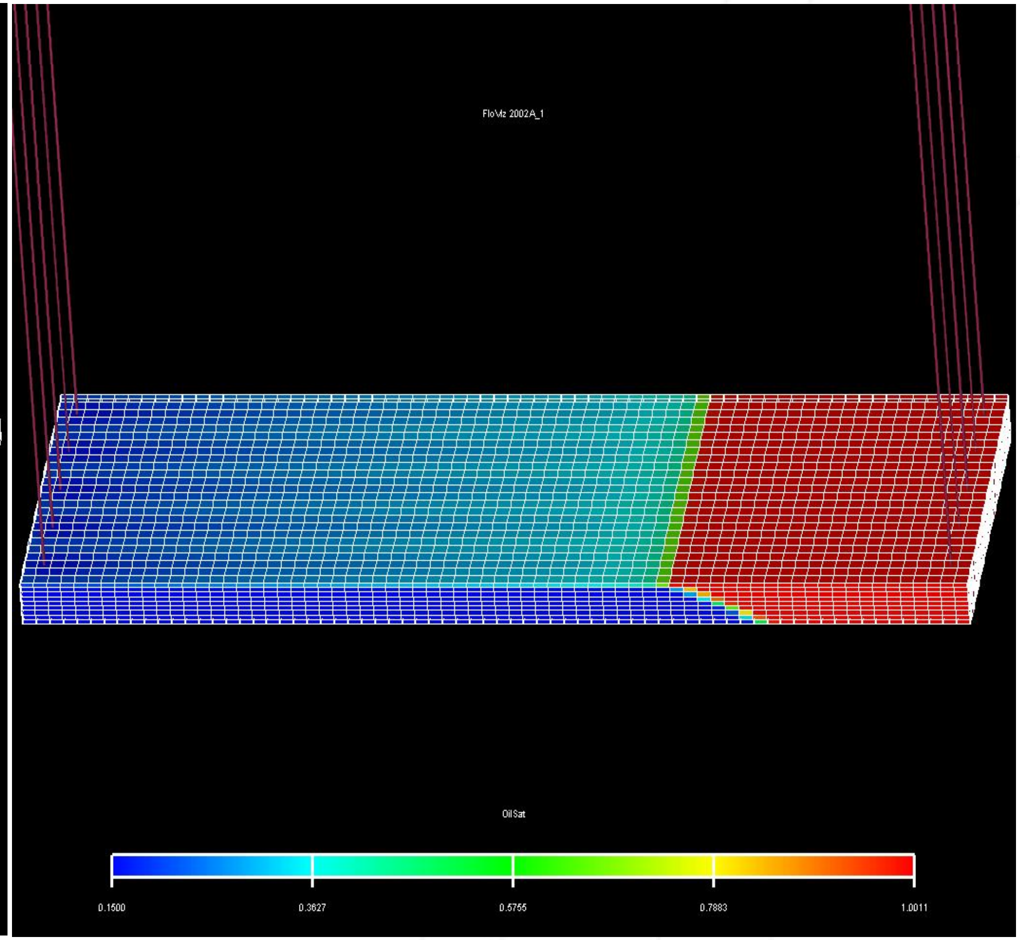


Does it Matter? – Simulated Production



Oil Saturation,
10 Material Regions and Averaged $C_r(p_f)$





Comments

- Rock Tables & Material Regions only needs redefining at *some* stress steps
  - ✓ Example run: All updates based on results from stress step no. 2

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☺ Improved reservoir simulator compaction functions reduces Visage run time:

- Example run, CPU time per stress step
- ➢ Iteration step 1: ~15 minutes
- ➢ Iteration step 2: 7-8 minutes
- ➢ Iteration steps 3 & 4: 1-2 minutes

Improved Coupling Scheme:

- Accuracy comparable to fully coupled
- Efficiency comparable to explicit coupled, often better (good predictor)
  - 1-10% of fully coupled run times

Conclusions

- By compaction of sand / sandstone
  - material grows stronger due to tighter packing
  - pore space is permanently deformed
  - ➢ Critical State Theory

- Disregarding stress state boundary effects (“arching”) can lead to grave errors, especially for weak materials

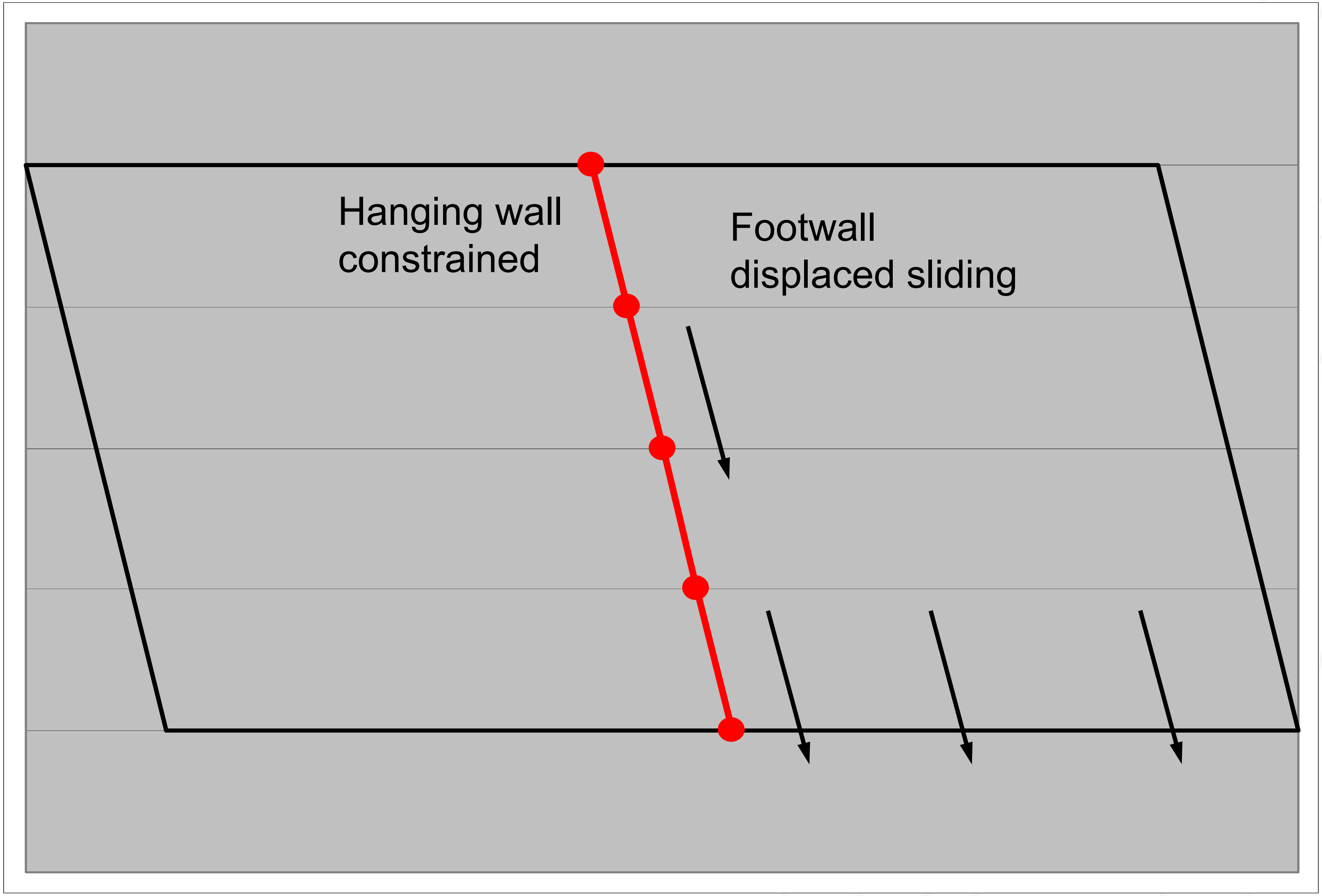
- Understanding compaction requires coupled simulations

- An improved coupling scheme has been presented

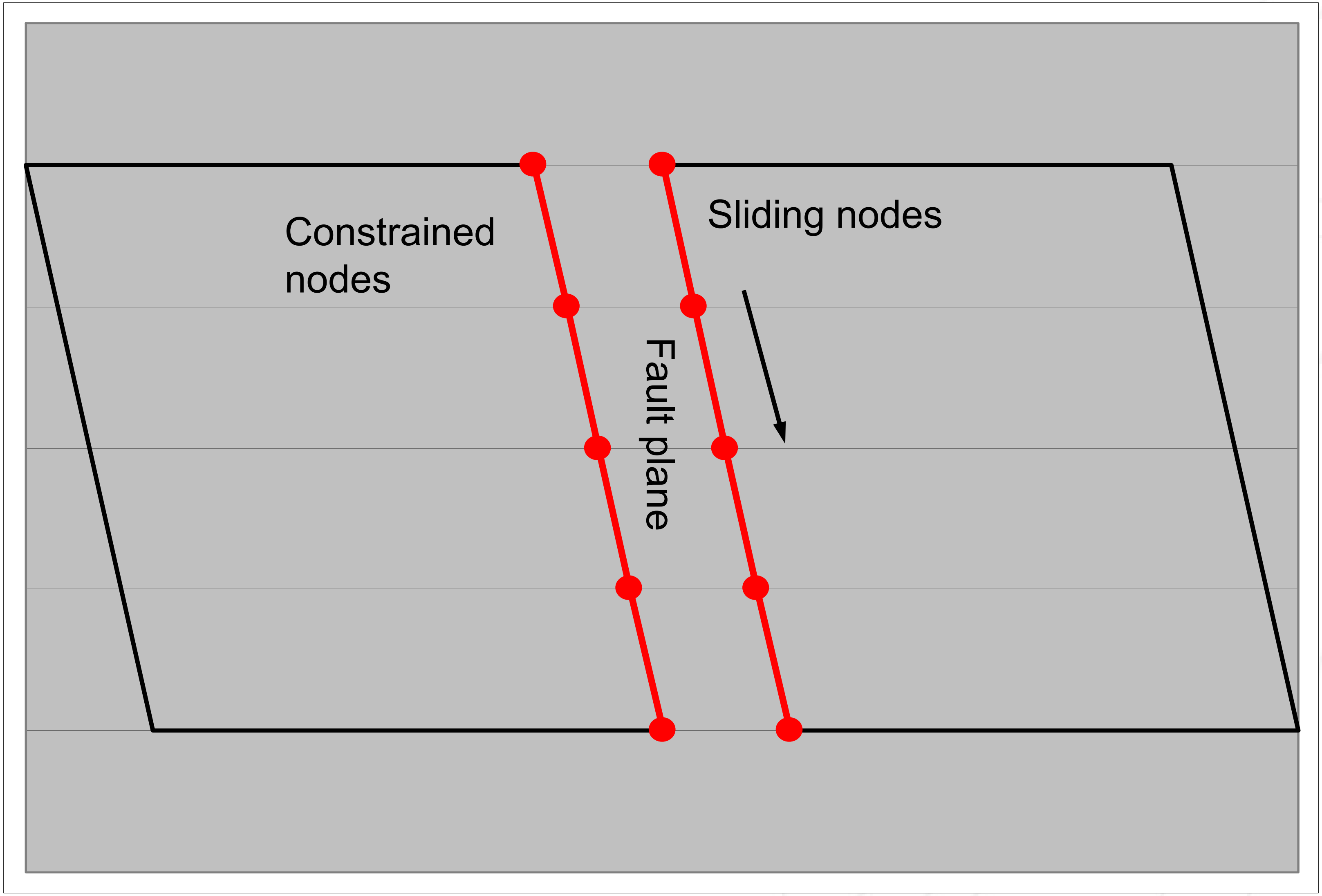
Simulation of Fault Propagation / Strength

Dynamic Sealing properties of Faults can be Modelled if the Stress Distribution Within the Fault is known at the start of Fluid Flow,
i.e. at the end of the Fault Generation process

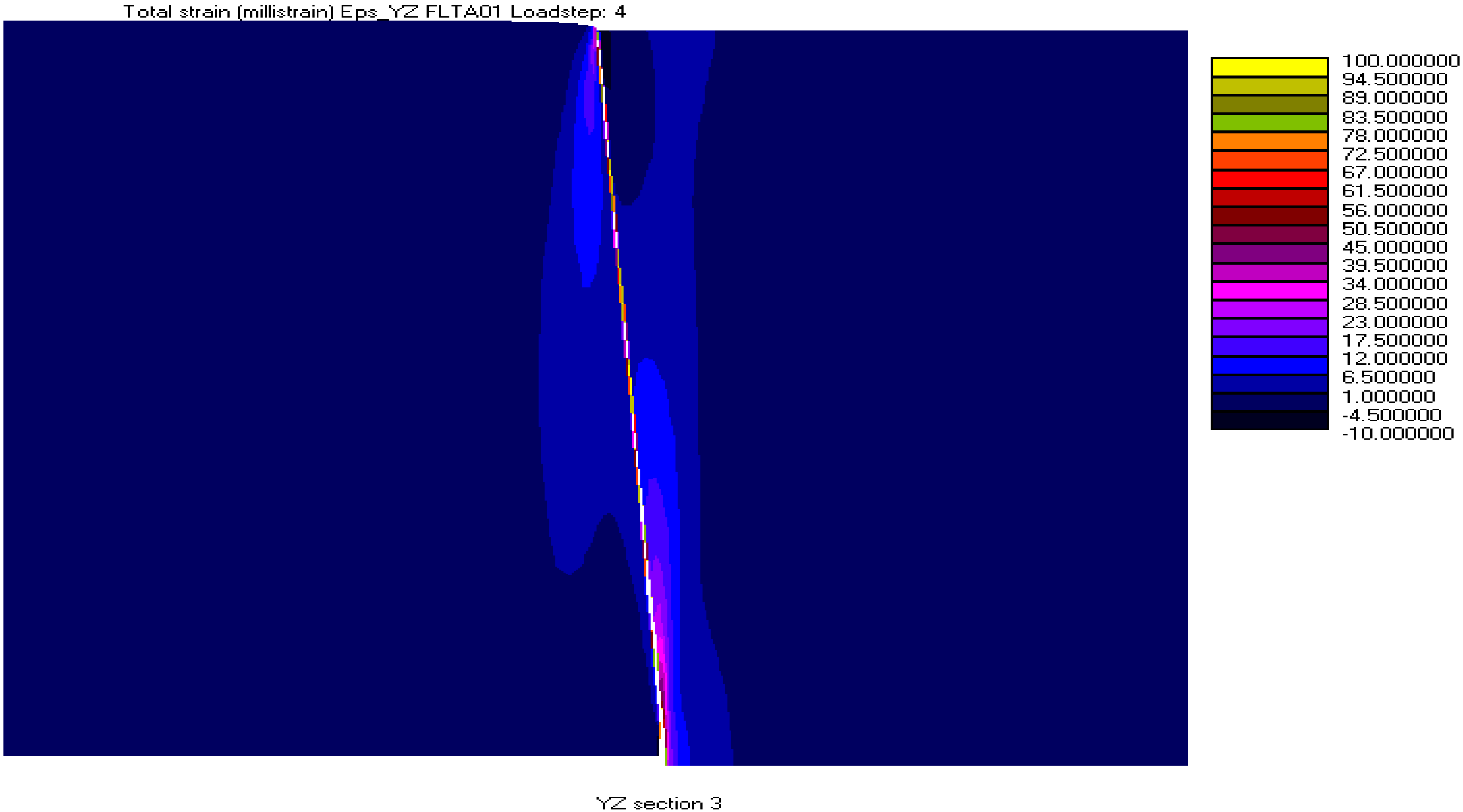
Zero-thickness fault, shared nodes



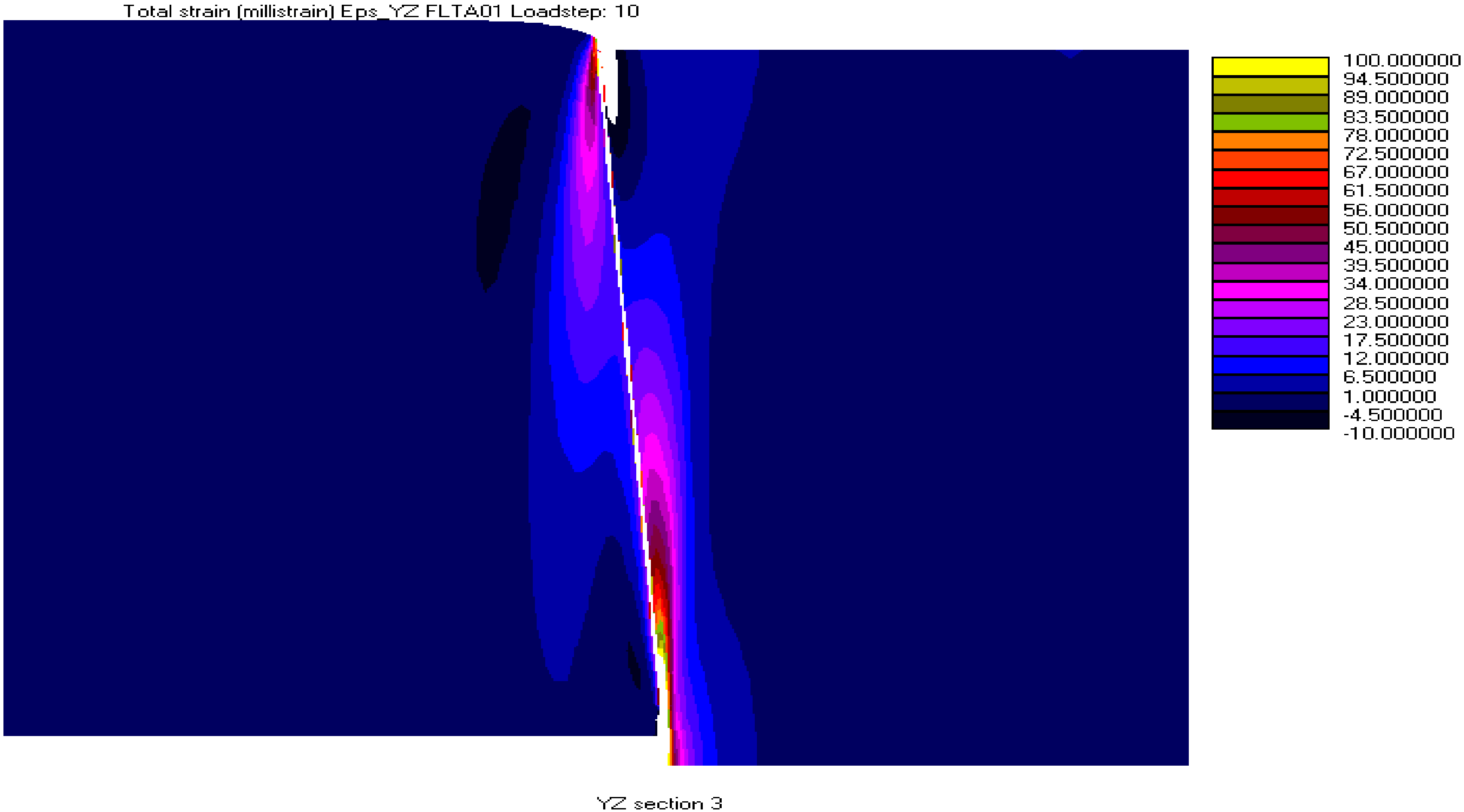
Fault volume, contact plane displacement



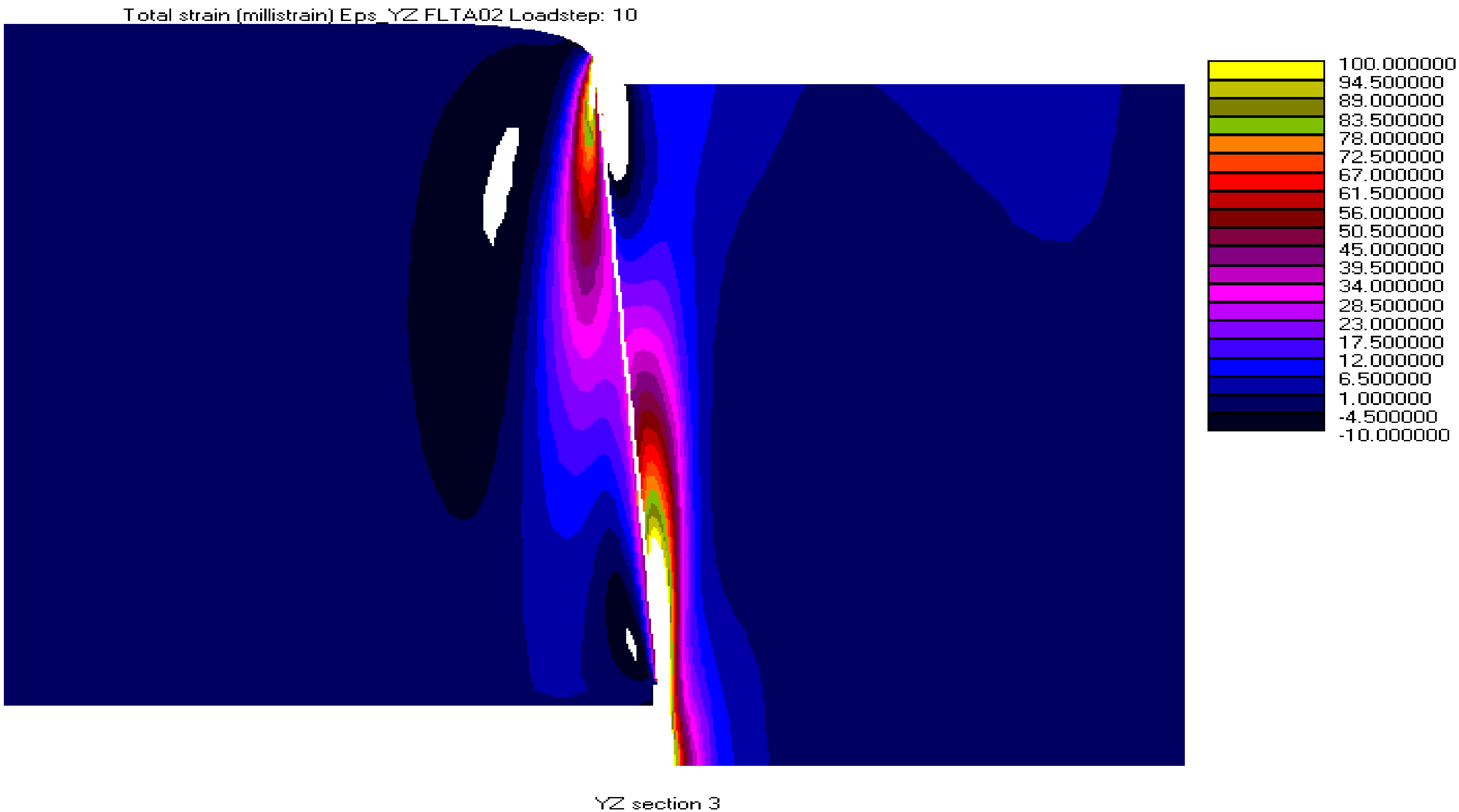
ε_{yz}, vertical displacement 4m



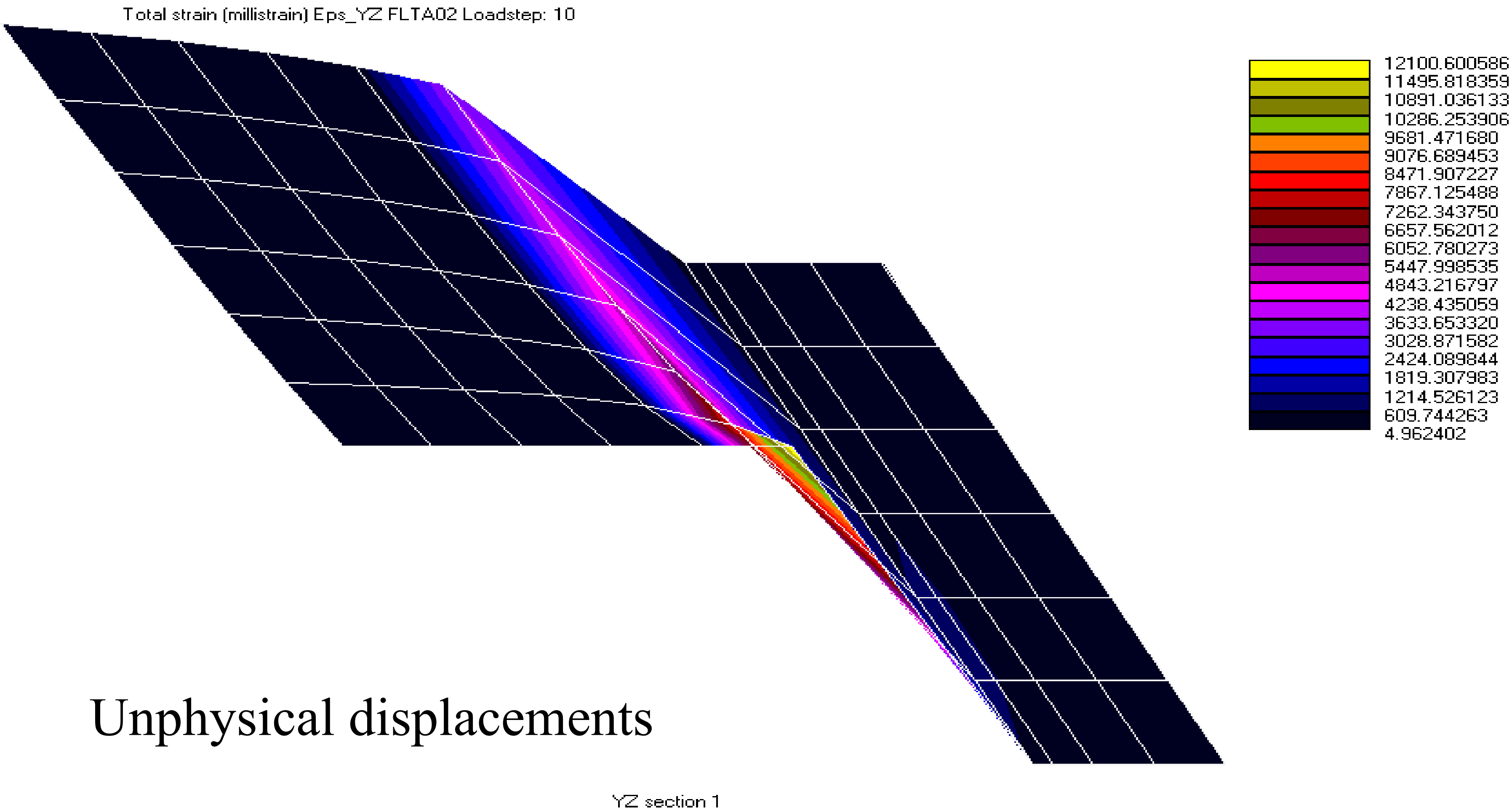
ε_{yz}, vertical displacement 10m



ε_{yz}, vertical displacement 20m

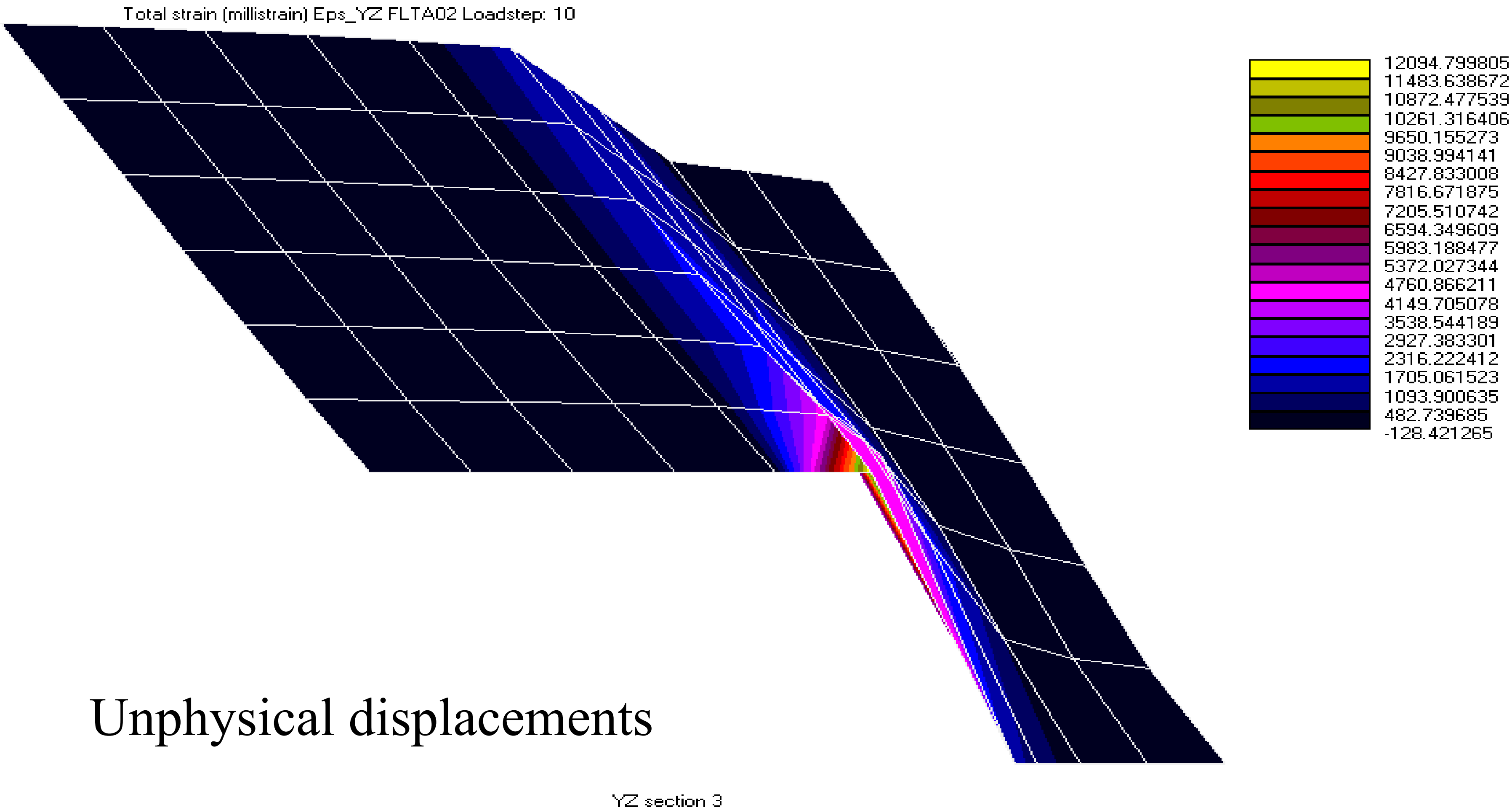


ε_{yz}, vertical displacement 20m, zoomed



Unphysical displacements

ε_{yz}, vertical displacement 20m, zoomed



Unphysical displacements

Unsolved issues

- Adaptive mesh, regenerated at each load step
- Pseudo-initialise with interpolated stress state from previous load step
- Automatic mesh refinement
- Handling of slip contact planes (surfaces) (elastic OK)
- Modelling of fault volume, fracturing